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Article

Model and Measure the Volatility of the Iraqi Dinar's Exchange Rate Against the US Dollar Using the AR(1)-GARCH(1,1) Model Under the t Student's Distribution for the Period 2015–2024

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Abstract: This study aims to model and measure the volatility of the Iraqi dinar's exchange rate against the US dollar for the period 2015–2024, using modern standard methods for financial time series. The research problem stems from the inadequacy of traditional linear models, such as Ordinary Least Squares (OLS), and the assumption of a normal distribution in capturing the extreme shocks experienced by the Iraqi economy. To overcome this limitation, the study employed an autoregressive model with conditional heteroscedasticity (AR(1)-GARCH(1,1)) model, assuming a Student's t-distribution. The results of the statistical tests showed a decisive rejection of the normality hypothesis based on the Jarque–Bera test, and provided mathematical evidence of the presence of a clustering phenomenon in volatility based on the ARCH-LM test. The estimation results also confirmed that the probability distribution of the dinar exchange rate returns is characterised by heavy tails with a degree of freedom of (ν=5.24), implying a high probability of severe shocks. The study concludes that conditional variance models are the most effective in measuring the exchange rate risk of the Iraqi dinar.

Keywords: GARCH models, heteroscedasticity, volatility pooling, fat tails, Student's t-distribution

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1. Introduction

The study of financial time series is one of the most complex fields in econometrics, given its dynamic nature and its susceptibility to shocks and news. In the Iraqi economy, the exchange rate of the Iraqi dinar against the US dollar represents the most important macroeconomic indicator, as it reflects the state of monetary and financial stability in a rentier economy that is highly sensitive to political and economic fluctuations [1].

Many studies have relied on classical linear regression models, assuming homoscedasticity and a normal distribution of the data, however, practical experience has shown that large shocks are not normally distributed but rather create what is known as the 'fat tails' phenomenon, and that fluctuations tend to cluster at specific time intervals, leading to a breakdown of the assumptions underlying the ordinary least squares (OLS) method [2].

The research problem lies in the urgent need for advanced statistical tools capable of managing these violent fluctuations and measuring them mathematically with high

precision, whilst avoiding underestimation of the true magnitude of the risks that financial decision-makers may face.

The research therefore aims to construct a statistical model to measure the volatility of the Iraqi dinar exchange rate returns for the period 2015–2024, overcoming the limitations of classical models. To achieve this objective, the research methodology is based on the GARCH family of models, specifically the hybrid model (AR(1)-GARCH(1,1)), whilst completely abandoning the assumption of a normal distribution in favour of the Student's t-distribution to ensure that extreme shocks are accommodated in full compliance with the axioms of probability (Kolmogorov's axioms).

2. Materials and Methods

This research adopted a quantitative econometric approach to model and measure the fluctuations of the Iraqi dinar/US dollar exchange rate series. To achieve its objectives, the applied aspect was implemented through a sequential process, beginning with data preparation. This involved transforming the daily exchange rate time series into logarithmic yields to minimize significant data gaps. The series was then subjected to three tests to determine the modeling path. The Dickey–Fuller test was used to establish the stationarity of the series. This was followed by the ARCH–LM test to detect the presence of conditional heterogeneity. Finally, the Jarque–Bera test was used to examine the probability distribution of the time series and determine whether it followed a normal distribution.

Based on the results of the three tests, which demonstrated the limitations of classical linear models (OLS), a hybrid model was designed. This model consists of an autoregressive mean equation using AR(1) and a conditional variance equation using GARCH(1,1). To ensure the model's ability to accommodate exchange rate shocks, the normality assumption was abandoned, and a t-Student distribution was used instead. Therefore, the parameters of the final model were estimated, and from the estimated parameters, the final mathematical equations for future prediction were extracted.

3. Results

3. Stochastic Processes and Time Series

The exchange rate of the Iraqi dinar against the US dollar does not move in fixed increments that can be predicted with high accuracy; rather, it is governed by a complex interplay of political and economic influences and is therefore characterised by mathematical randomness, which is known as a stochastic process [3].

A stochastic process is defined as a family of random variables $\{X_t, t \in T\}$, where t denotes time, defined on a probability space; in this study, the random variable represents the series of logarithmic returns on the exchange rate [4].

The aim of studying financial time series is not to eliminate randomness; rather, the primary objective is to decompose the stochastic process into two components.

The first component is the structural component, which represents the mean and is the part that can be predicted based on past information from the series.

The second component is the random error, which represents new information and news that reaches the market unexpectedly.

As random errors are not evenly distributed over time, the study of the variance and volatility of random processes is more important than the study of their means [5].

4. Kolmogorov's Axioms of Probability

For any standard economic model to be valid and applicable, it must not contradict the fundamental and essential principles of probability theory, as formulated by the Russian mathematician Andrei Kolmogorov in 1933 [6].

This research is based on two of Kolmogorov's axioms.

Axiom 1: The non-negativity axiom

The essence of this axiom states that the probability of any event occurring must be equal to or greater than zero ($P(A) \geq 0$).

In financial applications, this condition translates to the requirement that the conditional variance must always be positive ($\sigma_t^2 > 0$) [7].

When data are subject to severe shocks, some simple linear models fail to satisfy this condition; therefore, the approach taken in this research was to use Generalised Autoregressive Conditional Heteroscedasticity (GARCH) models, as their mathematical structure relies on the squared values ($\epsilon_{t-1}^2, \sigma_{t-1}^2$) This ensures that the variance remains positive at every time point, which is fully consistent with Kolmogorov's positivity axiom.

Axiom 2: the axiom of total certainty

This axiom states that the sum of all possible probabilities must equal one (P(Ω) = 1).

That is, the area under the probability distribution curve equals one. In stable markets, the probability mass is concentrated in the centre (as represented by the normal distribution), but when exchange rate jumps occur (economic crises), the probability mass shifts from the centre to the tails [8],

and consequently, the use of the normal distribution fails to measure the tails accurately, thereby violating the axiom of total certainty. Therefore, the t-distribution (Student's t-distribution) is used to account for the probability mass concentrated at the tails, thereby ensuring that the area under the curve remains equal to one.

5. Clustering of volatility

Financial time series are characterised by the phenomenon of clustering of volatility, which the mathematician Mandelbrot described as the tendency for small changes to be followed by small changes, and large changes to be followed by large changes, whether positive or negative.

In the Iraqi economy, this phenomenon is clearly evident; when a crisis occurs, the foreign exchange market experiences violent fluctuations that persist for long periods.

This behaviour violates one of the key assumptions of the least squares method, namely the assumption of homoscedasticity; consequently, what is known as conditional heteroscedasticity arises, necessitating a shift to models capable of capturing the temporal clustering of risks [9].

6. The phenomenon of fat tails

Classical financial models assume that asset returns follow a normal distribution with a skewness of zero and a kurtosis of 3; under this assumption, the probability of any fluctuations in exchange rates is mathematically negligible [10].

The actual reality of the Iraqi foreign exchange market proves the opposite; that is, sharp spikes and shocks occur more frequently than would be expected from a normal distribution. This is known as 'fat tails', meaning a distribution with high skewness. To address this issue, studies replace the normal distribution with distributions capable of accommodating such fluctuations; one of the most prominent of these is the t-distribution (Student's t-distribution), which is characterised by its degrees of freedom: the lower the degrees of freedom, the thicker the tails [11].

7. Models (AR-GARCH)

To address the combination of volatility and fat tails, a dual framework combining mean modelling and variance modelling was adopted, consisting of two components

First: the conditional mean equation (AR model)

The autoregressive (AR) model is used to explain the current value of a time series based on its previous (past) values [12].

In this study, the first-order AR(1) model was employed

$$r_t = \mu + \emptyset_1 r_{t-1} + \epsilon_t \quad (1)$$

Where

r_t: the return on the series

μ: the general constant

$\emptyset_1$: the autocorrelation parameter

ϵ_t: the random error

t: time

Secondly: The Conditional Variance Equation (GARCH Model)

Generalised conditional variance models were developed by Bollerslev to address the phenomenon of volatility clustering. The GARCH(1,1) model is based on the assumption that the current variance (σ_t^2) depends on the squares of past shocks and past variances [13].

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (2)$$

Where

ω: the constant term of volatility (and ω must be > 0)

α_1: a GARCH parameter that measures the market's sensitivity to shocks and new information (ϵ_{t-1}^2)

β_1 : a GARCH parameter that measures the persistence of volatility based on previous variance (σ_{t-1}^2)

To ensure the stability of the model, it is required that the sum of the two parameters be ($\alpha_1 + \beta_1 < 1$); when this sum approaches exactly one, this indicates very high persistence of shocks in the foreign exchange markets.

8. Description of Data and Mathematical Transformations

First: Source and Nature of the Data

The research is based on daily data on the exchange rate of the Iraqi dinar against the US dollar for the period 2015–2024; this data was obtained from the Central Bank of Iraq. This specific time period was chosen because it is punctuated by crises (severe shocks), making it an ideal period for testing the ability of standard models to capture volatility.

Second: Mathematical Transformations

A common error in financial econometrics is the direct application of models to raw price time series.

As exchange rates are often volatile, this leads to the problem of spurious regression.

To overcome this problem and prepare the data for modelling, the raw daily exchange rate series was transformed into a continuous logarithmic return series using the following equation

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (3)$$

Where

r_t: the logarithmic return on the exchange rate on day t

P_t: the exchange rate on day t

P_{t-1} : the exchange rate on the previous day, $t-1$

ln : the natural logarithm

9. Data suitability testing

Before commencing the estimation of the model parameters, the series of exchange rate returns must be subjected to a set of diagnostic tests. These tests aim to verify the statistical properties of the series and to confirm the presence of standard issues that justify the use of an advanced AR-GARCH model rather than simple ordinary least squares (OLS) regression models.

Three tests were conducted on the data

Augmented Dickey–Fuller (ADF) test

The aim of this test is to verify the stationarity of the exchange rate returns time series and to ensure that it is free from the unit root problem, thereby avoiding spurious regression [14].

To conduct the test, the following two hypotheses were formulated:

H_0: The time series contains unit roots

H_1: The time series does not contain unit roots

The probability distribution for this test is a specific distribution known as the Dickey–Fuller distribution. The calculated test statistic was (–12.45), and the absolute value of this statistic exceeds all tabulated values for the various significance levels (0.01, 0.05 and 0.1). Therefore, the decision is to reject the null hypothesis and accept the alternative hypothesis, i.e. that the series does not contain unit roots (the time series is stationary).

ARCH-LM Test

The aim of this test is to detect the presence of conditional heteroscedasticity (to determine whether market volatility occurs in waves or is stable) [15].

To conduct the test, the following two hypotheses were formulated:

H_0: The variance of the errors is homogeneous and stable over time

H_1: The variance of the errors is heterogeneous and stable over time

The probability distribution for this test is the chi-square distribution. The calculated test statistic was 48.72. Comparing this with the critical value of 3.84, and given that the calculated value is greater than the critical value, the decision is therefore to reject the null hypothesis and accept the alternative hypothesis, i.e. that the time series does not contain unit roots.

Jarque–Bera Test

The aim of this test is to examine the nature of the probability distribution of the time series and to ascertain whether it follows a normal distribution [16].

To conduct the test, the following two hypotheses were formulated:

H_0: The data follow a normal distribution

H_1: The data do not follow a normal distribution

The probability distribution for this test is the chi-square distribution, and the calculated test statistic was 15,420.3. Comparing the calculated test statistic with the critical value from the table at a significance level of 0.05, which is 5.99, and given that the calculated value is greater than the critical value, the decision is to reject the null hypothesis and accept the alternative hypothesis; that is, the data do not follow a normal distribution.

10. Estimation of the model parameters (AR(1)-GARCH(1,1))

Based on the results of the diagnostic tests, which confirmed the presence of ARCH effects and heavy tails in the data, the ordinary least squares (OLS) method was ruled out due to its inefficiency in handling heteroscedasticity; therefore, the maximum likelihood estimation (MLE) method was used to estimate the parameters of the combined model using the (Student's t-distribution).

The degrees of freedom for the Student's t-distribution were recorded as (ν=5.24), which is statistically significant; this constitutes conclusive evidence of heavy tails.

Table 1. presents the estimation results for the mean equation

Parameter		Coefficient	Std. Error	P-Value
Constant	μ	0.00012	0.00004	0.002**
AR(1)	$\emptyset_1$	0.154	0.042	0.000**

** indicates statistical significance at the 0.05 level

Table (1) shows that the autocorrelation coefficient ($\emptyset_1$ = 0.154) is positive and highly significant. This indicates the presence of short-term memory in exchange rate returns.

Table 2. presents the estimation results for the conditional variance equation

Parameter		Coefficient	Std. Error	P-Value
Constant	ω	0.00003	0.00001	0.001***
ARCH	α_1	0.125	0.028	0.000***
GARCH	β_1	0.842	0.035	0.000***

*** indicates statistical significance at the 0.01 level

It can be seen from Table (2) below that:

1. The positive axiom: the estimated parameters (β_1, α_1, ω) are all positive and greater than zero; this ensures that the conditional variance (σ_t^2) will always be positive, thereby satisfying Kolmogorov's positive axiom.

2- Market sensitivity to shocks (α_1) : The ARCH coefficient was 0.125; this parameter measures the extent to which today's volatility is influenced by shocks (unexpected news) that occurred the previous day.

3- Long memory of volatility (β_1): The GARCH parameter recorded a high value of (0.842); this figure confirms a high persistence of risk, as crises that drive up the exchange rate do not fade quickly but continue to have an impact on the market for extended periods.

4- Variance stability and volatility aggregation: the sum of the two parameters (α_1 , β_1)

$\alpha_1 + \beta_1 = 0.967$

Given that the sum is close to, but less than, exactly one, this demonstrates two things: firstly, that shocks have a long-lasting effect that fades slowly; and secondly, that the model maintains its statistical stability.

11. Mathematical Forecasting of Exchange Rates

Having confirmed the efficiency of the AR(1)-GARCH(1,1) model and its success in diagnostic tests, the estimated parameters are used to derive the expected future values of both the exchange rate return and its conditional variance, and subsequently to estimate the expected actual exchange rate of the Iraqi dinar.

The forecasting system is constructed in three sequential steps to estimate the future value

Step 1: Forecasting the logarithmic return. Based on the estimated mean equation parameters, the expected return for period (t+1) is calculated using the information available for the current period (t), as shown in the equation.

$$\hat{r}_{t+1} = \mu + \emptyset_1 r_t \quad (4)$$

Substituting the estimated values from Table (1) into Equation (4)

$$\hat{r}_{t+1} = 0.00012 + 0.154\, r_t \quad (5)$$

Equation (5) means that the expected return at time (t+1) is based on the path of the realised return at time (t).

Step 2: Forecasting conditional variance. Forecasting volatility is the most important feature of GARCH models. The variance for period (t+1) is forecast based on the parameters of the estimated variance equation, as shown in the equation.

$$\hat{\sigma}^2_{t+1} = \omega + \alpha_1 \epsilon_t^2 + \beta_1 \sigma_t^2 \quad (6)$$

Substituting the estimated values from Table (2) into Equation (6)

$$\hat{\sigma}^2_{t+1} = 0.00003 + 0.125\, \epsilon_t^2 + 0.842\, \sigma_t^2 \quad (7)$$

Equation (7) is not used to forecast the exchange rate directly, but rather to generate dynamic confidence intervals; when a market crisis occurs, the value of (ϵ_t^2) increases, causing the confidence interval for the next forecast to widen automatically a warning signal to the decision-maker of increased risk. In times of stability, the confidence interval narrows, thereby increasing the accuracy of the forecast.

Step 3: Deriving the actual forecast price. Given that the model is based on a series of logarithmic returns, the financial decision-maker does not benefit from the return figure as much as from the actual exchange rate derived from it; Therefore, the exponential function is used as the mathematical inverse of the natural logarithm to convert the expected return ($\hat{r}_{t+1}$) into the expected actual price ($\hat{P}_{t+1}$), using the latest available actual price (P_t). This is done by taking the exponential of Equation (3)

$$\hat{P}_{t+1} = P_t\ e^{\hat{r}_{t+1}} \quad (8)$$

Substituting equation (5) into equation (8) gives

$$\hat{P}_{t+1} = P_t\ e^{0.00012+0.154\, r_t} \quad (9)$$

Equation (9) provides the expected value of the actual exchange rate for the following day based on the exchange rate for the current day,

4. Discussion

Based on the tests carried out on the return series for the exchange rate of the Iraqi dinar against the US dollar, a number of have been reached

1. The exchange rate returns of the Iraqi dinar against the US dollar do not follow a normal distribution; this demonstrates that the market is subject to severe shocks, and therefore any assumption of a normal distribution would be misleading when measuring risk.

2. Based on the value of the degrees of freedom for the Student's t-distribution, which was $\nu = 5.24$, it was found that the probability distribution of exchange rate returns is characterised by fat tails.
3. The ARCH-LM test, which yielded a value of 48.72, confirmed the presence of conditional heteroscedasticity. This implies that fluctuations in the foreign exchange market do not occur randomly but rather in waves, with large shocks followed by further large shocks and periods of stability followed by further stability.
4. The study demonstrated that the use of the autoregressive integrated conditional heteroscedasticity (AR (1)-GARCH(1,1)) model with a Student's t-distribution is the optimal and most accurate standard choice for capturing the dynamics of the Iraqi market, outperforming ordinary least squares (OLS) models, which fail in the face of the problems of heteroscedasticity and autocorrelation.
5. Based on the ADF test, which yielded a value of (–12.45), it was confirmed that the exchange rate return series is stationary, which lends high reliability to the results.
6. The model's outputs demonstrated that forecasting conditional variance provides an assessment of the magnitude of future risks in the foreign exchange market.

5. Conclusion

This research aimed to study and analyze the fluctuations in the exchange rate of the Iraqi dinar against the US dollar. The research concluded that exchange rate risk changes over time and is strongly affected by shocks. Furthermore, the research demonstrated the model's effectiveness in providing an accurate mathematical measure for calculating the expected future price of the Iraqi dinar based on the cumulative memory of exchange rates.

The researchers recommend that, when studying time series of the Iraqi exchange rate, researchers should cease to assume a normal distribution as a given and instead move towards using modified distributions such as the Student's t-distribution. Researchers are recommended to conduct future studies based on non-symmetric GARCH models, such as EGARCH or TGARCH, to determine whether the Iraqi market reacts to bad news with a greater intensity than it does to good news. It is recommended that financial decision-makers adopt the conditional variance forecasting equation as a proactive early-warning system in the management of the foreign exchange market.

REFERENCES

[1] C. Gourieroux and J. Jasiak, *Financial Econometrics: Problems, Models, and Methods*, 2018.
[2] S. G. Chortane and K. Naoui, "Information entropy theory and asset valuation: A literature survey," *International Journal of Accounting, Business and Finance*, vol. 2, no. 1, pp. 42–60, 2022.
[3] K. Al-Daffaiea, M. S. AL-Musaylh, Q. R. M. Al-Faisal, and Q. M. Shallal, "Monthly exchange rate prediction based on artificial intelligence models and Iraqi Dinar against United States dollar," in *AIP Conference Proceedings*, vol. 3232, no. 1, p. 030001, Oct. 2024.
[4] G. A. Pavliotis, *Stochastic Processes and Applications*, vol. 60. Texts in Applied Mathematics, 2014.
[5] P. Doukhan, *Stochastic Models for Time Series*, vol. 80. New York, NY, USA: Springer, 2018.
[6] A. N. Kolmogorov, "The theory of probability," in *Mathematics, Its Content, Methods, and Meaning*, vol. 2, pp. 110–118, 1963.
[7] M. Foster and L. MacLean, *Quantitative Methods in Risk Analysis: A Practitioner's Guide*, vol. 22. World Scientific, 2026.
[8] K. Mukhia, I. Ansari, S. R. Luwang, and M. Nurujjaman, "Core-periphery dynamics in market-conditioned financial networks: A conditional P-threshold mutual information approach," *arXiv preprint* arXiv:2601.00395, 2026.
[9] A. Gaunersdorfer, C. H. Hommes, and F. O. Wagener, "Bifurcation routes to volatility clustering under evolutionary learning," *Journal of Economic Behavior & Organization*, vol. 67, no. 1, pp. 27–47, 2008.
[10] C. Adcock, M. Eling, and N. Loperfido, "Skewed distributions in finance and actuarial science: A review," *The European Journal of Finance*, vol. 21, nos. 13–14, pp. 1253–1281, 2015.
[11] K. Sano, "A binary decision model and fat tails in financial market," *Applied Sciences*, vol. 12, no. 14, p. 7019, 2022.

[12] J. Kaur, K. S. Parmar, and S. Singh, "Autoregressive models in environmental forecasting time series: A theoretical and application review," *Environmental Science and Pollution Research*, vol. 30, no. 8, pp. 19617–19641, 2023.
[13] L. Bauwens, S. Laurent, and J. V. Rombouts, "Multivariate GARCH models: A survey," *Journal of Applied Econometrics*, vol. 21, no. 1, pp. 79–109, 2006.
[14] E. Paparoditis and D. N. Politis, "The asymptotic size and power of the augmented Dickey–Fuller test for a unit root," *Econometric Reviews*, vol. 37, no. 9, pp. 955–973, 2018.
[15] H. Yıldırım and F. V. Bekun, "Predicting volatility of bitcoin returns with ARCH, GARCH and EGARCH models," *Future Business Journal*, vol. 9, no. 1, p. 75, 2023.
[16] C. M. Jarque, "The Jarque-Bera test," in *International Encyclopedia of Statistical Science*, pp. 1271–1273. Berlin, Heidelberg: Springer, 2025.