

Econometric Analysis of Agricultural Food Products Using the Cobb-Douglas Production Function (Using the Example of Jizzakh Region)

Bekzod Sultonmurodovich Toshpulov

Tashkent State University of Economics
Senior Lecturer, Department of Econometrics
Email: btoshpulov@gmail.com

Abstract: This article examines the monitoring and trends of agricultural food production in the region. A statistical assessment of the analysis of agricultural food production in the Jizzakh region using the Cobb-Douglas function is considered. Based on the research results, the possibility of econometric analysis using standard deviation and oscillation deviation has been developed. Analysis of specific results of the Cobb-Douglas model. Graphs and models were compiled using values from 2010 to 2025.

Keywords: Mean deviation, fluctuation coefficient, correlation coefficient, food, production, determination coefficient, Student t-statistic criterion, Fisher's criterion, Cobb-Douglas production function, linear regression analysis.

Introduction

In agricultural food production, the production function ensures a quantitative correlation between costs and production. Economists prefer to use the form of the Cobb-Douglas production function, where is output, is capital, and is labor, because it provides a relatively accurate description of the economy and is known to be very easy to work with algebraically. The popularity of this feature is actually responsible for easy analytical processing and satisfies the constraints imposed on the production function. These restrictions include:

- both economic factors are important for production, so no product can be produced without these two variables;
- every expenditure produces a positive product;
- as the utilization of each expenditure increases, the surplus product decreases;
- additional product produced from one expenditure;

Also, each farm or cluster is very cautious about the future of producers in order to make the right decisions related to what products to produce, how much to produce and how to produce them. All these decisions are directly related to the cost considerations and market conditions in which agricultural food producers must operate. In this regard, this study should help propose the most appropriate functional form of the production process for the food-producing sectors of agriculture in the Jizzakh region. In this study, we will empirically analyze the Cobb-Douglas production function for the region's agricultural food production sector.

K.Hagendorf shows that the Cobb-Douglas function is the only production function characterized by the constant functional distribution of income from production factors [1]. He explained that assessing the parameters of the gross output function plays an important role in increasing growth and productivity.

Researchers have extensively applied the Cobb-Douglas production function in the fields of agriculture and food production [2]. In his research, Theodore W. Schultz conducted research on human capital and labor productivity in agriculture, as well as labor productivity relative to land and capital.

Zvi Griliches empirically assessed the effects of land, labor, capital, and technology (breeding, fertilizers) in agriculture in the United States (mainly in corn cultivation) through the production function and used the Cobb-Douglas production function [3]. Along with this, the Russian scientist (economist) Yuri Vasilyevich Yaroshenko analyzed the efficiency of using agricultural resources (land, labor, and capital factors) between 1950 and 1970 [4].

Research Robert Evans and Douglas Nort (1950-1960) collected and analyzed data on US agriculture in the late 19th and early 20th centuries. In the analysis, the scope of influence of these factors was studied using the agricultural production function, which is close to the Cobb-Douglas form [5]. Land (L) – land area used in agriculture, Capital (C) – machinery, equipment, irrigation systems, Labor (N) – the number of workers employed in agriculture. In their scientific research, A.L. Kehindi and T.T. Awoyem analyzed the issue of increasing the economic efficiency of sawn timber in the states of Oyo and Osun in southwestern Nigeria. Cobb-Douglas used the stochastic boundary method to estimate the production function, thereby estimating technical, distributional, and economic efficiency [6]. The study shows that, on average, saw mills in the states of Oyo and Osun have high technical, distribution, and economic efficiency. In both cases, they argued, machinery has the potential to increase output using the same level of cost or reduce the current level of cost to produce at the same level.

Since the 1980s, World Bank and FAO researchers have evaluated the land-labor-capital ratio in developing countries (India, Brazil, African countries) by constructing empirical models using the Cobb-Douglas function [7], [8]. Researcher Eugene Stiglitz, a World Bank economist, used the Cobb-Douglas model to estimate the land-labour-capital contribution to India's agricultural output of wheat and rice in the 1980s [9]. Hans Binswanger and Gershon Feder studied the effects of irrigation, capital, and labor on productivity in India and Africa between 1985 and 1986. M.M. Hossain. A.K. Majumder and T. Basak, on the other hand, saw an increasing interest in entrepreneurs and the theory of food production in order to make the right decisions about what products, how much and how to produce [10]. The researchers wanted to propose the most appropriate functional form of the production process for large manufacturing industries in Bangladesh. In the study, they calculated the Cobb-Douglas production function using the terms additional error and multiplicative error [11]. The main objective of the study was to select the Cobb-Douglas production model to measure the production process of some selected manufacturing industries in Bangladesh [11]. The researchers constructed various models and used sampling criteria to compare the Cobb-Douglas production function with the additional error duration.

Wu.M.D. used the indirect least squares estimating normal distribution of the Cobb-Douglas production function coefficients [12]. The researcher developed a model that combines a Cobb-Douglas type function with simultaneous multiplicative and additive errors. The specification was a natural generalization of "pure" models with added or multiplied stochastic terms. The researcher used the maximum probability approach to evaluate the Cobb-Douglas type model, including both multiplicative and additive errors. Used a maximization algorithm due to the complexity of the probability function. Additionally, macroeconomic growth perspectives and production dynamics have been broadly evaluated by Lucas [13], Nishi [14], and Madsen [15].

Methodology

Based on the scientific and methodological approaches outlined above, a comprehensive empirical analysis was conducted based on official data from the Jizzakh Regional Statistics Department. According to it, a complete statistical database has been compiled and implemented into the analytical process regarding the volume of agricultural food production for the period 2010–2025, employment indicators in the sector (labor force size), land resources involved in agriculture, and the main elements of production capital (technical and technological means).

The Cobb-Douglas production function is expressed in the form of a stochastic (probability) model as follows.

$$Y = f(K, L, N) = AK^{\alpha}L^{\beta}N^{\lambda}e^M$$

Here, Y is the volume of food production in agriculture (tonnes), K is capital (the number of technical means), L is the area of land used in agriculture, N is the number of people employed in agriculture (employees, managers, business managers), M - is the stochastic error limit, e - is the base of the natural logarithm, A is a constant coefficient, α - capital elasticity, β - land elasticity, λ - population elasticity (labor).

As can be seen from the equation, there is a nonlinear relationship between the production of agricultural food products and the three costs. However, we can calculate this model by converting it to a linear model with natural logarithms:

$$\ln Y = \ln A + \alpha \ln K + \beta \ln L + \lambda \ln N$$

$$\ln Y = B + \alpha \ln K + \beta \ln L + \lambda \ln N \quad \text{here } B = \ln A$$

Thus, the written model with parameters B, α , β and λ is linear, and therefore it is a linear regression model. It should be noted that Y and K, L, N were nonlinear in endogenous and exogenous variables, but after natural logarithm these variables are linear. In short, equation is a log-log, linear model of two-sided variables. *adqiqot metodologiyasi qishloq xo'jaligi oziq-ovqat mahsulotlari baholashda bir qator statistik va ekonometrik usullarga tayangan.* An empirical analysis was conducted to study the (partial) elasticity of production relative to α capital investment, β the elasticity of the employed population, and the λ partial elasticity of agricultural land areas relative to food products.

Analysis and results

In this study, we will examine data on the empirical probability of Fisher's criterion (F-distribution), Student's t-statistic, Darwin-Watson test (DW), and Breusch-Pagan test assumptions for the classical linear regression model of agricultural food products in the Jizzakh region. The created econometric model is expressed through the Cobb-Douglas production function, which depends on the volume of agricultural food production, the number of technical equipment, the area of land used, and the number of people employed in agriculture. The model was calculated based on 26-year observation data.

Table 1. Economic indicators of agricultural food production in the Jizzakh region.

Years	Comparative assessment of agricultural food production in billion soums (Y)	Number of agricultural machinery (K)	of Area of land used in agriculture, hectares (L)	Number of population employed in agriculture,
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				thousand (N)	people
2000	946,59	9316	473884	141.5	
2001	1023,20	9423	473985	141.9	
2002	1055,96	9612	474789	142.3	
2003	1152,40	9023	475739	144.4	
2004	1284,16	8916	475985	148.9	
2005	1290,94	8856	476572	150.1	
2006	1298,23	9010	478181	154.6	
2007	1302,54	9211	479931	158.7	
2008	1315,59	10326	482096	159.1	
2009	1363,94	10456	482748	159.2	
2010	1847,32	10612	482603	159.6	
2011	2827,04	10746	483657	166.1	
2012	3471,79	10896	483131	168.6	
2013	4039,68	10948	482922	176.9	
2014	5072,64	11023	481122	187.0	
2015	6066,22	11079	480487	192.1	
2016	7257,21	12600	479641	195.3	
2017	9144,23	12842	479140	195.7	
2018	11413,66	13144	477693	190.5	
2019	14347,56	13929	479552	194.5	
2020	16794,33	12755	486226	199.5	
2021	21330,89	12991	483787	230.0	
2022	23657,51	12203	480780	234.8	
2023	28093,23	12389	500957	236.1	
2024	28516,56	12647	501408	243.3	
2025	36460,10	14316	503235	249.6	

In Table 1, to construct a linear regression of data on the production of agricultural food products in the Jizzakh region for the period 2000-2025, it is necessary to logarithm the data due to the fact that the units of measurement are different. The Cobb-Douglas production function always contains linear regression equations that are analyzed.

Table 2. Logarithmic values of economic indicators for agricultural food production in the Jizzakh region.

Years	Comparative assessment of agricultural production in soums LN(Y)	Number of agricultural machinery LN(K)	Area of land used in agriculture, hectares LN(L)	Number of population employed in agriculture, thousand LN(N)	people in LN(N)
2000	6,83	9,1395	13,0687	4,9523	
2001	6,90	9,1509	13,0689	4,9551	
2002	6,93	9,1708	13,0706	4,9579	

2003	7,02	9,1075	13,0726	4,9726
2004	7,13	9,0956	13,0731	5,0033
2005	7,14	9,0889	13,0744	5,0113
2006	7,14	9,1061	13,0777	5,0408
2007	7,15	9,1282	13,0814	5,0670
2008	7,16	9,2424	13,0859	5,0695
2009	7,16	9,2549	13,0873	5,0702
2010	7,16	9,2697	13,0869	5,0727
2011	7,58	9,2823	13,0891	5,1126
2012	7,78	9,2962	13,0880	5,1275
2013	7,94	9,3009	13,0876	5,1756
2014	8,17	9,3077	13,0839	5,2311
2015	8,34	9,3128	13,0826	5,2580
2016	8,52	9,4415	13,0808	5,2745
2017	8,78	9,4605	13,0797	5,2766
2018	9,04	9,4837	13,0767	5,2497
2019	9,26	9,5417	13,0806	5,2704
2020	9,40	9,4537	13,0944	5,2958
2021	9,62	9,4720	13,0894	5,4381
2022	9,75	9,4094	13,0832	5,4587
2023	9,89	9,4028	13,1243	5,3850
2024	9,95	9,4026	13,1252	5,4232
2025	10,46	9,4246	13,6362	5,6947

In Table 2, we analyze the volume of food products in the Jizzakh region using natural logarithm values: LN (Y), agricultural machinery LN (K), land area used in agriculture LN (L), and the number of people employed in agriculture LN (N).

The Cobb-Douglas production function was calculated in the R-program based on logarithmic values of data on the volume of agricultural food products and the factors influencing it for the period 2000-2025.

$$A = e^{\alpha} = e^{-1.5254} = 0.2175 \quad \alpha = \ln A,$$

$$\ln Y = -1.5254 + 1.8446 \ln K + 1.0584 \ln L + 6.3998 \ln N$$

We study the direct linear regression model as a log-log model. In the model, a 1% increase in K (number of agricultural vehicles) means that the volume of agricultural food products will increase by 1.84% when other factors remain unchanged. If an increase in L (agricultural land area) by 1% leads to an increase in agricultural food production by 1.058%, while other factors remain unchanged. N means that when the number of people employed in agriculture increases by 1%, agricultural food production increases by 6.3998%, while other factors remain unchanged. $\alpha = 1.8446$ $\beta = 1.0584$ $\gamma = 6.3998$

Table 3. Coefficients of the regression equation obtained using the R-program.

	Coef.	St.deviasion.
Constanta	-1.5254	54,995

lnK	1.8446	0,6164
lnL	1.0584	4,2435
lnN	6.3998	0,6381

We will analyze how the sum of the coefficients of these factors affects the volume of agricultural production.

$$1.8446 + 1.0584 + 6.3998 = 9.3028 > 1$$

The sum of the total coefficients of the Cobb-Douglas production function is greater than 1, and the average coefficient influence of the factors is 9.3028. The volume of agricultural production in the Jizzakh region is influenced by factors such as the number of people employed in agriculture, the number of technical equipment, and the area of land used.

Table 4. Results of data correlation analysis using the R-program.

	LnY	LnK	LnL	LnN
LnY	1.000			
LnK	0.922	1.000		
LnL	0.697	0.586	1.000	
LnN	0.976	0.694	0,647	1.000

Based on Table 4, it was established that the correlation between the primary factor Y and the influencing factor K was 92.2%, the correlation between Y and L was 69.7%, and the correlation between Y and N was 97.6%.

Let's check the adequacy of the model using Fisher's F-test. $F=0.0000$ (p-value), which indicates the adequacy of the model.

Table 5. Results of checking the general statistical adequacy of the model using the Fisher criterion.

```

> #####
> # 4. FISHER F-TEST
> #####
>
> alpha <- 0.05
>
> F_real <- summary(model)$fstatistic[1]
> df1 <- summary(model)$fstatistic[2]
> df2 <- summary(model)$fstatistic[3]
>
> F_crit <- qf(1 - alpha, df1, df2)
> p_F <- pf(F_real, df1, df2, lower.tail = FALSE)
>
> cat("F haqiqiy =", F_real, "\n")
F haqiqiy = 125.4523
> cat("F kritik =", F_crit, "\n")
F kritik = 3.072467
> cat("p-value (F) =", p_F, "\n")
p-value (F) = 1.442939e-13
> #####

```

Table 5 shows that the table value of F-criterion is 3.072467, and the calculated (actual) value of F -criterion is 125.4523. Indicates the adequacy of the constructed model. The content of the H_0 hypothesis is that all deviation coefficients are simultaneously equal to zero: $\beta_K = \beta_L = \beta_N = 0$. The alternative hypothesis H_1 means that at least one coefficient is non-zero, so the model is considered adequate.

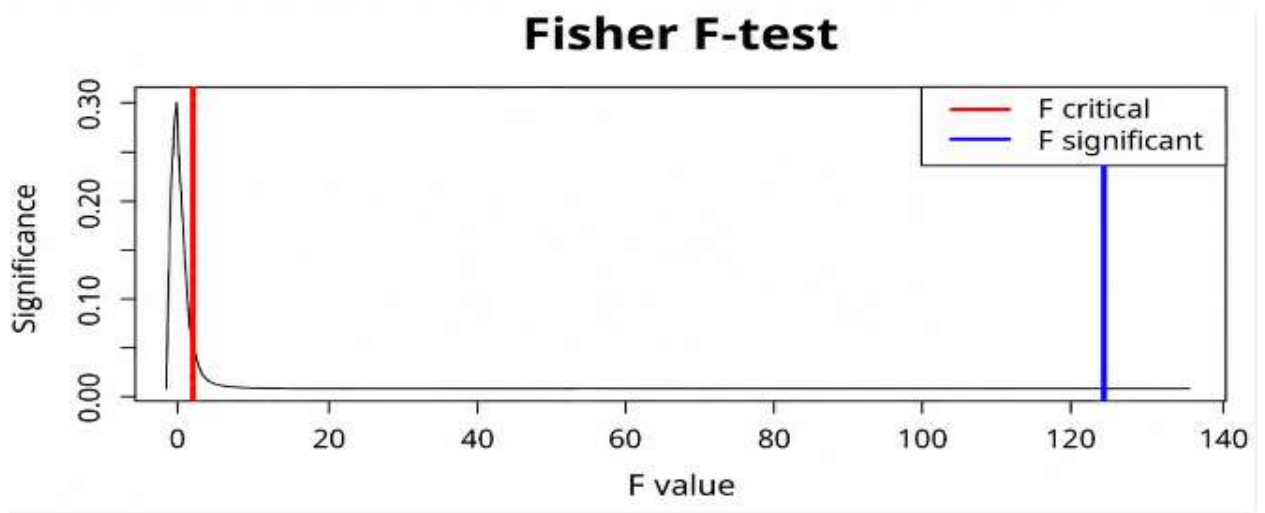


Figure 1. Graph verifying the adequacy of the Cobb-Douglas production model using the F-criterion

The statistical significance of the parameters of the Cobb-Douglas production function is evaluated using the Student's t-statistic criterion.

Table 6. The result of checking the statistical significance of the model using Student's t-test

```

Console Terminal Background Jobs
R - R 4.4.3 - ~/
> # 6. STUDENT t-TEST
> #####
>
> t_real <- summary(model)$coefficients[, "t value"]
> df_t <- df.residual(model)
> t_crit <- qt(1 - alpha/2, df_t)
> p_t <- summary(model)$coefficients[, "Pr(>|t|)"]
>
> cat("t haqiqiy qiymatlar:\n")
t haqiqiy qiymatlar:
> print(t_real)
(Intercept)          lnK          lnL          lnN
-1.525415      1.844674      1.058479      6.399836
>
> cat("t kritik =", t_crit, "\n")
t kritik = 2.079614
> cat("p-value (t):\n")
p-value (t):
> print(p_t)
(Intercept)          lnK          lnL          lnN
1.420759e-01 7.923768e-02 3.018612e-01 2.413209e-06
>

```

In Table 6, the Student's t-statistic criterion is 2.0796 at a table value of $P=0.05$. According to the obtained result, since $\ln K$ (number of agricultural machinery) is equal to $t=1.844674 < 2.0796$, the significance level indicates statistical significance at a critical point of 0.1 percent. $\ln L$ (agricultural land area) - $t=1.0584 < 2.0796$ is considered insignificant in the t-statistic criterion at 0.05 percent. $\ln N$ (number of employed population) - $t=6.3998 > 2.0796$ is considered statistically significant if it exceeds the critical point by 0.05 percent in the t-statistic criterion. The number of people employed in agriculture has passed a very good t-statistic criterion, and this factor yields an analysis result as the main driver factor for the resulting factor.

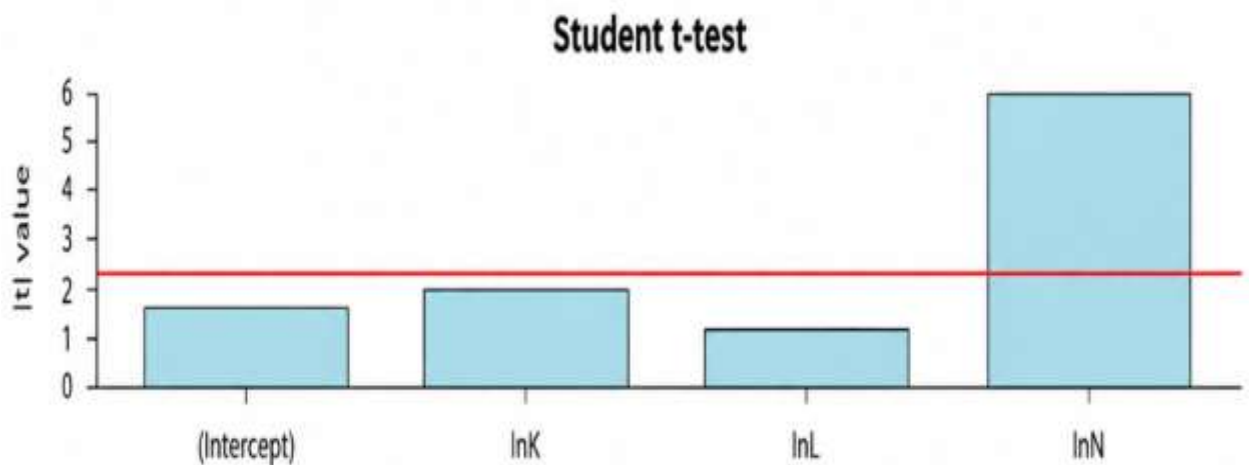


Figure 2. Representation of P values using the t-statistic criterion of the Cobb-Douglas production model coefficients.

We check for the presence of autocorrelation in the residuals of the equation using the Durbin-Watson (DW) criterion.

Table 7. Result of the test for autocorrelation based on the Durbin-Watson (DW) criterion.

```

Console Terminal Background Jobs
R - R 4.4.3 - ~/
as.Date, as.Date.numeric

>
> dw <- dwtest(model)
> DW_real <- dw$statistic
> p_DW <- dw$p.value
> cat("Durbin-watson =", DW_real, "\n")
Durbin-watson = 0.5188198
> cat("p-value (DW) =", p_DW, "\n")
p-value (DW) = 2.709683e-07
>
> #####
> # 9. DURBIN-WATSON GRAFIGI
> #####
> curve(dnorm(x, mean = 2, sd = 0.5),
+       from = 0, to = 4,
+       main = "Durbin-watson testi",
+       xlab = "DW qiymat")
>
> abline(v = DW_real, col = "blue", lwd = 2)
> abline(v = 2, col = "red", lwd = 2)
>
> legend("topright",
+       legend = c("DW haqiqiy", "Ideal DW = 2"),
+       col = c("blue", "red"),
+       lwd = 2)

```

Using the data in Table 7, the calculated value of the Durbin-Watson (DW) criterion is DW=0.5188198, indicating the presence of autocorrelation.

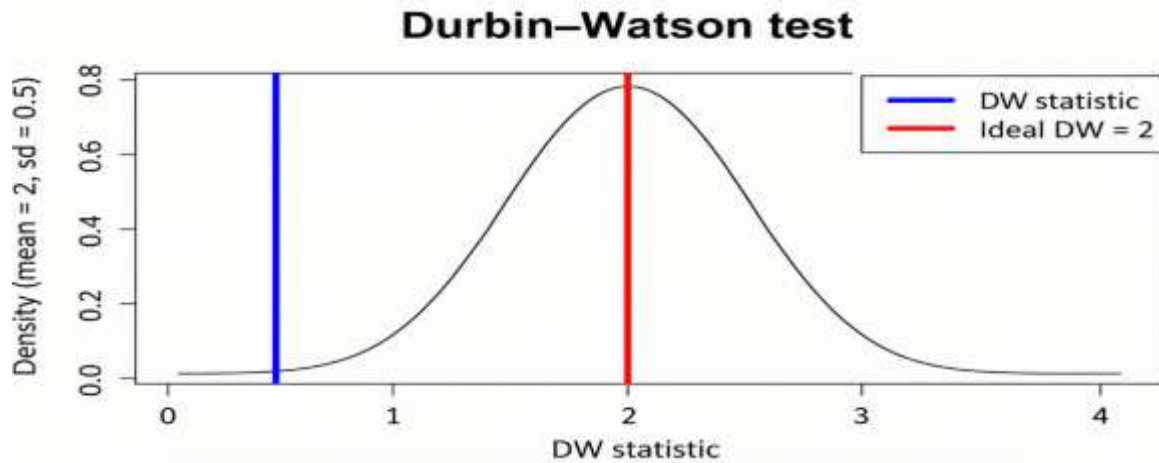


Figure 3. Visual representation of the autocorrelation determined based on the Durbin-Watson (DW) criterion.

In Figure 3, we can see that there is an autocorrelation in the visual, and this red line is not close. We will conduct another test to verify the Cobb-Douglas production function, namely the heteroskedasticity test. When checking for heteroskedasticity, the results of the Breusch-Pagan test are evaluated. To perform the Breusch-Pagan test using the least squares method, we perform the following operations.

1. Residuals are determined using the least squares method.

$$Y = \alpha_0 + \alpha_1 K_i + \alpha_2 L_i + \alpha_3 N_i + m_i \quad (1)$$

the remainders are taken from the least squares.

$$\hat{\varepsilon}_i = Y_i - \hat{Y}_i \quad (2)$$

2. Calculate the squares of the remainders.

$$u_i = \hat{\varepsilon}_i^2 \quad (3)$$

3. We write the regression equation of endogenous and exogenous factors, which explains the squares of these residues.

$$u_i = a_0 + a_1 K_i + a_2 L_i + a_3 N_i + m_i \quad (4)$$

4. From this regression equation, we calculate R .
5. LM – using this formula, we study the agricultural data of the Jizzakh region step-by-step and perform χ^2 an asymptotic distribution of the $(n \cdot R^2)$ statistics.

$$LM = n \cdot R^2$$

where is the n number of observations;

LM has an asymptotic distribution $\chi^2(k)$, where (k) is the number of factors ($k = 3$). The Breusch-Pagan test has two calculation methods, namely the Breusch-Pagan LM (Lagrange Multiplier) and Breusch-Pagan (F) tests, which we will calculate using these $(p > 0.05)$ tests (Table 7) and must be less than. The coefficient of determination was calculated to be $R^2 = 0.947$.

Breusch-Pagan (LM): $\chi^2 \approx 0.391, p \approx 0.942$ ($K \cdot L, K \cdot N, L \cdot N$) - compared to the distribution was not rejected, but the heteroscedasticity was rejected.

Breusch-Pagan (F): $F \approx 0,111$, $p \approx 0,958517$ - were not rejected compared to the distribution, and these values indicate that since heteroskedasticity and homoskedasticity are on the boundary of the hypothesis, we can conclude that the model passes the test.

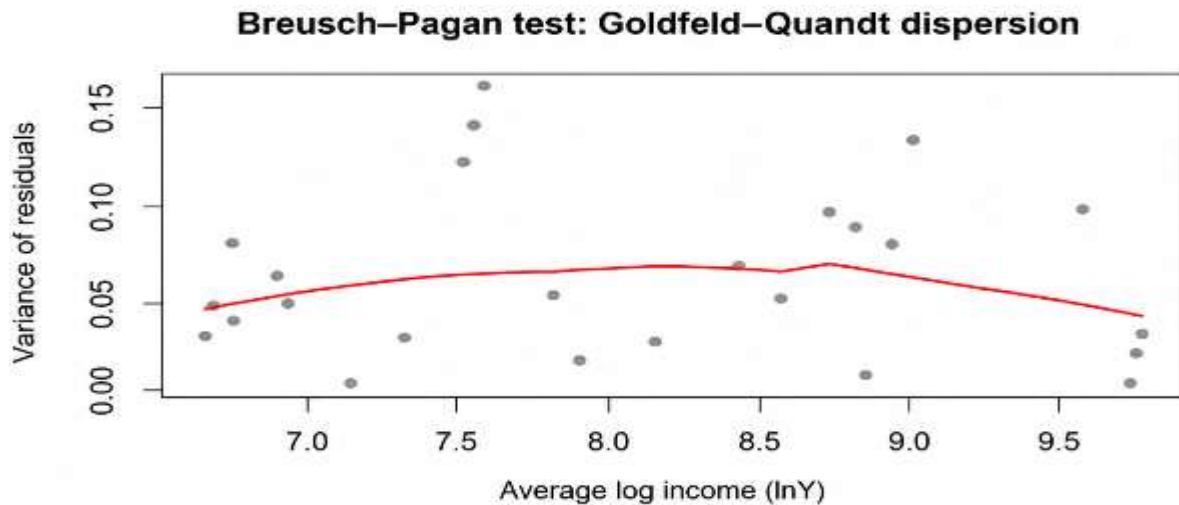


Figure 4. Graphical representation of the Breusch-Pagan test

As seen in Figure 4, the green line in the Breusch-Pagan test was not rejected when we tested the hypothesis, indicating that it is stable and the variance is constant. However, when the hypothesis is tested, the residual variance does not depend on linear factors. The dots in the figure are the squares of the real residuals. when checked for error, was approximately equal to.

Thus, the test results show that the residual variance is stable, and there is no heteroscedasticity in the hypothesis of the Breusch-Pagan (F) regression equation. The Breusch-Pagan (LM) test also rejected the heteroscedasticity of the hypothesis. It was found that the hypothesis contains a homoskedasticity. The Cobb-Douglas function model for agricultural food products in the Jizzakh region can be considered reliable.

Conclusion

The results of econometric modeling and the analysis of the developed linear regression equation allow for the following fundamental conclusion: first, it has been proven that the labor force employed in agriculture, developed land areas, and technical and technological means (capital) are the main driving factors determining the volume of production in the industry. Consequently, there are significant economic reserves for further expanding production volumes in the agricultural sector by increasing labor resource efficiency and modernizing the material and technical base in the future.

In the Cobb-Douglas production functions, the sum of the coefficients is 1. Since the determination coefficient in this equation is 0.94, it shows that it affects 94 percent. In conclusion, it should be noted that the agricultural sector of the Jizzakh region has great strategic potential in the production of food products, in particular, the optimal management of labor resources and the material and technical base (technical means) serves as the main factor in ensuring long-term and sustainable growth in the volume of production in the industry.

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