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Design and Simulation of Contact Lenses for Astigmatism Correction using ZEMAX

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Abstract: The research presents the design and subsequent simulation of toric contact lenses intended for the clinical correction of ocular astigmatism. The optical model was constructed in Zemax software, integrating a validated healthy eye model (Liou & Brennan) with the contact lens geometry to evaluate the resulting optical properties and aberrations. Optical performance was quantitatively evaluated using a suite of established image quality metrics: the modulation transfer functions (MTF) and point spread function (PSF), root mean square (RMS) spot size, and geometric encircled energy (EE). Simulation parameters were standardized to a 2.5 mm pupil diameter, a 5-degree field of view (FOV), and a spectral range of 455 nm to 655 nm.

Keywords: Eye Modeling, Astigmatism Correction, Toric-Lens, Contact Lens.

1. Introduction

Astigmatism is a common refractive error that affects vision quality, where refraction varies along the eye's meridians [1]. Instead of coming together at a specific focal point, the light beams entering the eye create focal lines [2]. To put it another way, astigmatism is a medical condition in which parallel light rays of the cornea do not focus on the retina [3]. Regular or irregular astigmatism are both possible. Astigmatism may have a corneal, lenticular, a retinal origin. Additionally, there are three types of astigmatism: simple, complex, and mixed [4]. can be treated using various methods including glasses [5], contact lenses [6], intraocular lenses [7], and surgery with laser techniques [8]. In this study, we systematically design and simulate toric contact lenses for a defined spectrum of corneal astigmatism, quantifying performance using standardized optical metrics. Toroidal, spherical, ellipsoidal, and sphero-cylindrical surfaces have been used for designing astigmatic corrective lenses [9], [10]. Recent studies found that toric contact lenses provide better visual acuity and contrast sensitivity than spherical contact lenses [11], while spherical lenses are not effective in masking corneal toricity during topography, whereas toric lenses can neutralize the corneal cylinder [12]. To optimize the non-spherical surfaces in contact lenses, include toroidal and asphere surfaces, we utilized Zemax software and optimization techniques. We analyzed the simulation results using MTF, PSF, and focal beam spot diagrams.

2. Materials and Methods

2.1. The Emmetropic Eye Model

The modeling of the human eye plays a fundamental role in vision science and ophthalmic care. Among existing models, the Liou-Brennan (L.&B.) [13] optical model has emerged as the preferred framework for simulating refractive errors. The eye is described by this model as a sphere-like structure in the total optical power at 60.35 D and an axial length in 23.95 mm. It incorporates detailed optical parameters such as refractive index, thickness, radius of curvature, and semi-diameter for the cornea, aqueous humor, and other ocular components, pupil, crystalline lens surfaces, vitreous body, and retina, thereby supporting accurate and physiologically realistic simulations. These values are systematically presented in Table1.

Table 1. Healthy Eye Parameters (Liou-Brennan eye model) [13].

Surf.	**type**	Comment	Radius. (mm)	Thickness. (mm)	Glass	Semi-Diameter	Conic
0	Standard	Object	Infinity	10^9	-	0	0.00
1	Standard	Input Beam	Infinity	500.000	-		
2	Standard	Anterior cornea	7.77	0.550	1.38, 50.23	5	-0.18
3	Standard	Posterior cornea	6.40	3.160	1.336,50.23	5	-0.60
4	Standard	Pupil	Infinity	0.000	1.336, 50.23	1.25	0
5	Gradient 3	Lens- front	12.40	1.590	Grad A		-0.94
6	Gradient 3	Lens- back	Infinity	2.430	Grad P	5	0.00
7	Standard	Vitreous	-8.10	16.239	1.336, 50.23	5	0.96
8	Standard	Retina	-12.00	-	-	5	0

A key feature of the Liou-Brennan model is its representation of the crystalline lens, which includes a gradient refractive index profile and is subdivided into anterior and posterior segments, each with distinct optical characteristics [13]. The following formulas provide the distribution in gradient index and also index in the anterior (n_A) as and posterior surface(n_P) [14]:

$[n_A(z,r)] = 1.368 + 0.049057z - 0.015427z^2 - 0.001978r^2,$ (1),

$[n_P\,(z,r)] = 1.4078 - 0.006605z^2 - 0.001978r^2,$ (2),

All eye media can be described by the dispersion relation because of the (L. & B.) model. $n(\lambda) = n_{555} + 0.0512 - 0.1455\lambda + 0.0961\lambda^2$ (3),

For this model, the z-axis was oriented along the visual axis of the eye model eye with cylindrical coordinates (z, r). The n_{555} is the material refractive index at 0.555μm incident wavelength. The simulation of a healthy eye model (L.& B.) using Zemax software is shown in Figure 1.

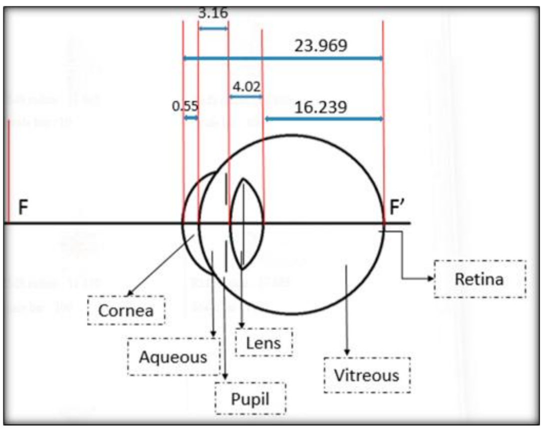


Figure 1. Schematic of a healthy eye (L.& B.) model [14].

2.2. Astigmatic Eye Parameters

This article focuses on the modelling and treatment of astigmatism. In an astigmatic eye, the cornea exhibits two distinct dioptric powers in perpendicular meridians. Regular astigmatism occurs when these principal meridians are oriented vertically and horizontally; otherwise, the astigmatism is classified as irregular. This study specifically addresses regular astigmatism of the anterior cornea, in limited by 2 D, where the radius [7.760, 7.80] determine the curvature and principal powers of 48.08 D and 49.21 D along the X and Y axes, respectively.

An eye suffering from astigmatism can be simulated using a toroidal surface in Zemax software [15], [16]. Details for this model are provided in Table 2. The surface's sag in the Y–Z plane is given by Equation (4) [16].

$$z = \frac{cy^2}{1+\sqrt{1-(1+k)c^2y^2}} + \sum_{i=1}^{N} \alpha_i \, y^{2i} \qquad (4)$$

The curvature and conic shape of the toroidal surface along the X and Y axes are determined by the parameters c and k, respectively.

Table 2. Parameters of astigmatic eye model.

Surf.	**type**	Comment	Radius. (mm)	Thickness. (mm)	Glass	Semi-Diameter	Conic
0	Standard	Object	Infinity	Infinity	-	0	0.00
1	Standard	Input Beam	Infinity	500.000	-		
2	Toroidal	Anterior cornea	Y-Radius: 7.6 X-Radius: 7. 8	0.550	1.38, 50.23	5	-0.18
3	Standard	Posterior cornea	6.40	3.160	1.336,50.23	5	-0.60
4	Standard	Pupil	Infinity	0.000	1.336, 50.23	1.25	0
5	Gradient 3	Lens- front	12.40	1.590	Grad A		-0.94
6	Gradient 3	Lens- back	Infinity	2.430	Grad P	5	0.00
7	Standard	Vitreous	-8.10	16.239	1.336, 50.23	5	0.96
8	Standard	Retina	-12.00	-	-	5	0

In this model, because the radii of curvature for the anterior cornea differ between the X and Y meridians, the refractive power must be calculated separately for each principal meridianby applying the classic lensmaker's equation (Equation 5) [17].

$$P = \frac{n_2 - n_1}{R} \qquad (5)$$

Where P is the refractive power $(unit: Diopters\ D)$, n_1 is the refractive index of the incident medium, n_2 is the refractive index of the exiting medium, and R is the radius of curvature $(unit: meters)$.

In this model, because the radii of curvature for the anterior cornea differ between the X and Y meridians, the refractive power must be calculated separately for each principal meridian. The refractive power can be calculated by using a refractive index of 1.0 for air (n_1) and 1.376 for the corneal front (n_2) as [18]:

X-Direction (Flat Meridian):

$$P_{ant,x} = \frac{1.376 - 1.000}{0.0078} = \frac{0.376}{0.0078} \approx 48.21\ D$$

Y-Direction (Steep Meridian):

$$P_{ant,y} = \frac{1.376 - 1.0}{0.0076} = \frac{0.376}{0.0076} \approx 49.47\ D$$

2.3. Design Biconic Contact Lenses to Correct Astigmatism

To simulate far, intermediate, and near vision distances within the software, multi-configuration settings were employed. These settings assigned three specific object distances infinity, 1000 mm, and 500 mm, as outlined in Table 3. This approach enabled an evaluation of how varying object distances affect the clarity and sharpness of vision achieved with the contact lenses. By modeling different viewing distances, valuable insights were obtained into the performance of both contact lenses in correcting astigmatism and ensuring clear, focused vision across a range of daily activities.

Table 3. Multi-configuration settings to simulate vision distances from distance to near.

Operands	surface	Config 1	Config 2	Config 3
MOFF	0	Distance	Intermediate	Near
THIC (mm)	0	10^9	1000.000	500.000
PRAM	4/3	0.000°	−10.000°	−20.000°

To design a contact lens, two new surfaces (2 and 3) were added in front of the cornea. The contact lens material is made of Poly (methyl methacrylate) (PMMA) reinforced with Titanium Dioxide nanoparticles (TiO_2 NPs) in 0.01 w/v concentrations and is characterized by high transparent (T > 90%), lightweight, and impact-resistant contact lens [19], with a refractive index and Abbe number of 1.602 and 31.0, respectively [20], and a thickness of 0.135 mm. The Zemax software was configured with a pupil diameter of 2.5 mm and a visual field of $(0°, 10°, 20°)$. In the visual area (photopic state), the retina's spectral sensitivity for light wavelengths in 0.47, 0.51, 0.555, 0.61, & 0.655 μm was taken into consideration.

Table 4. Information of contact lenses simulation.

Surf. type		Comment	Radius. (mm)	Thickness (mm)	Glass	Semi-Diameter	Conic
2	Biconic	C-lens-front	Y-Radius: 7.784 X-Radius: 7.799	0.135	PMMA-TiO_2	4.5	Y-Radius: - 0.190 X-Radius: -0.176
3	Even Aspherical	C-lens-back	7.700	0.05	1.336, 50.23	5.0	0

To simulate astigmatism correction, Zemax employs a contact lens featuring a biconic front surface. This particular surface is described as a curve on the Y-Z plane that is rotated around an axis that contacts the Z-axis and travels parallel to the Y-axis [16], [21].

$$z = \frac{c_x x^2 + c_y y^2}{1+\sqrt{1-(1+k_x)c_x^2 x^2-(1+k_y)c_y^2 y^2}} \tag{6}$$

Where $c_x = \frac{1}{R_x}$, $c_y = \frac{1}{R_y}$,

The primary goal was to correct the astigmatism in the affected eye [21]. After the condition was definitively identified, a merit function was employed as a quantitative measure of system performance. This function evaluates the correction by calculating the deviations between the current design parameters and their target values, and it is typically structured as follows:

$$(MF)^2 = \frac{\sum_i W_i\,(V_i - T_i)}{\sum_i W_i} \tag{7}$$

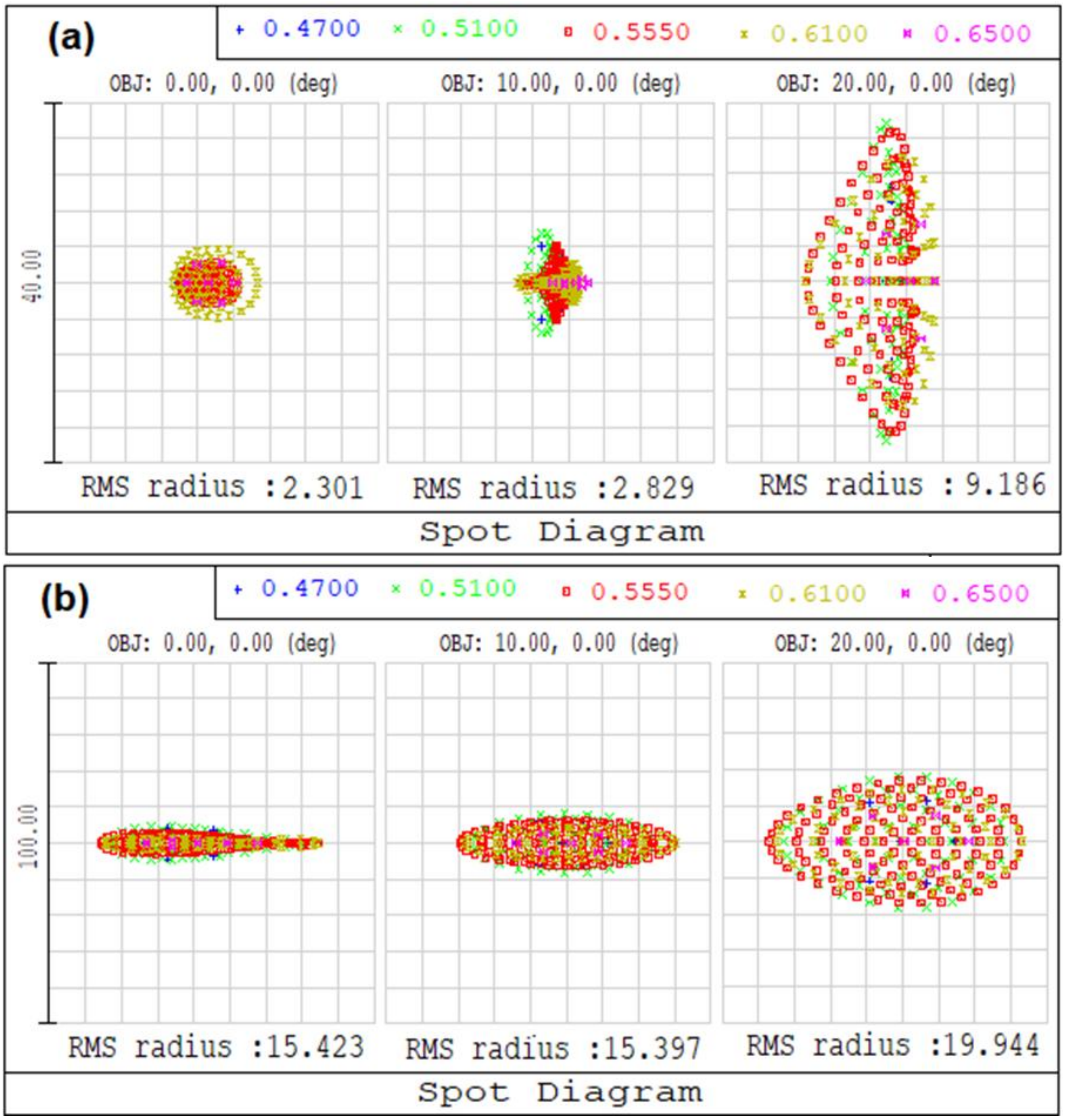
Zemax's optimization algorithms iteratively refine the design variables to minimize the merit function. The goal is to achieve the lowest possible merit function value, indicating the closest match to the design targets and, consequently, the best theoretical performance for the contact lens simulation, as shown in Table 4. For this specific analysis, optimization was guided by key operands: MTFA (diffraction MTF averaged from the sagittal and tangential components), STRH (Strehl Ratio, related to the point spread function), SPHA (spherical aberration), EFFL (effective focal length), ASTI (astigmatism), and RSCE (RMS spot radius relative to the geometric image centroid).

3. Results and Discussion

Spot diagrams offer valuable insight into the imaging quality of an optical system. By examining these diagrams, we can evaluate key factors such as aberrations, distortions, and overall image clarity. This analysis supports the refinement of the design and enhances system performance for improved visual outcomes.

Retinal spot diagrams in the 3 vision zones (0°, 10°, and 20°) were created after the optimization process, progressing from distance to near vision (left to right) within the patient's eye, and shown in **Figure 2.** A quantitative indicator for retinal image quality for astigmatic eyes, both before as well as after contact lens correction, is the root means square (RMS) value of the spot diagram.

The RMS value shows a large increase in aberrations before correction, which causes a defocused retinal image and a wide, erratic distribution of light intensity. The spot diagram shows a noticeable improvement at image quality following correction; aberrations are decreased, bringing the focus spot closer to a ideal point and reducing its radius. Astigmatism is successfully corrected by the contact lens, which significantly lowers aberrations and creates a more consistent, finely focused retinal image. A narrower and concentrated light distribution is reflected in the RMS value, which drastically drops.



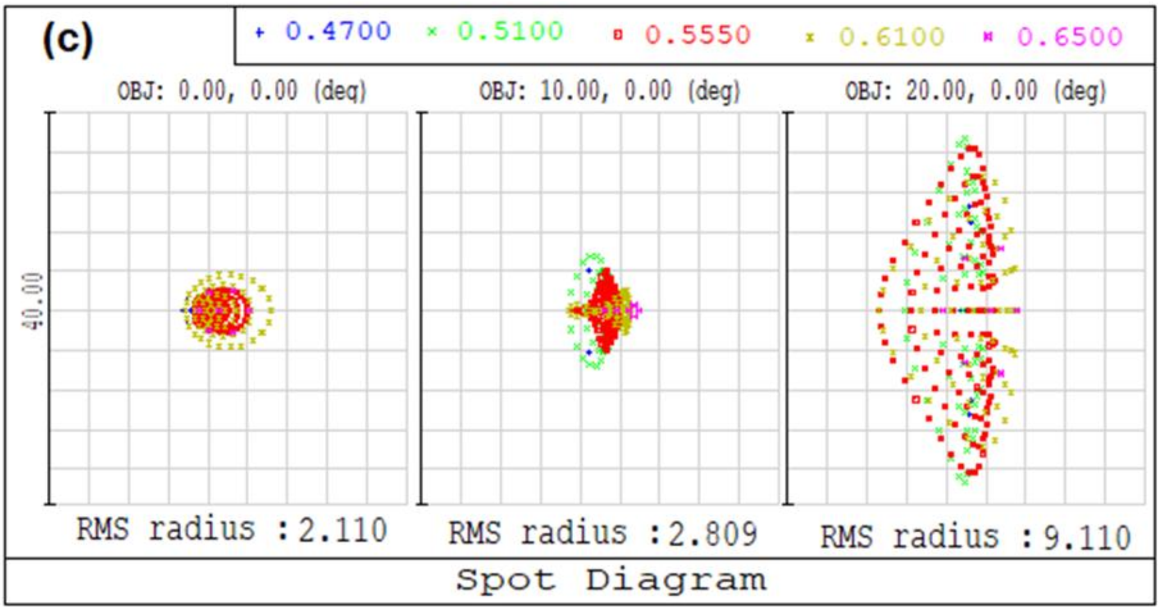
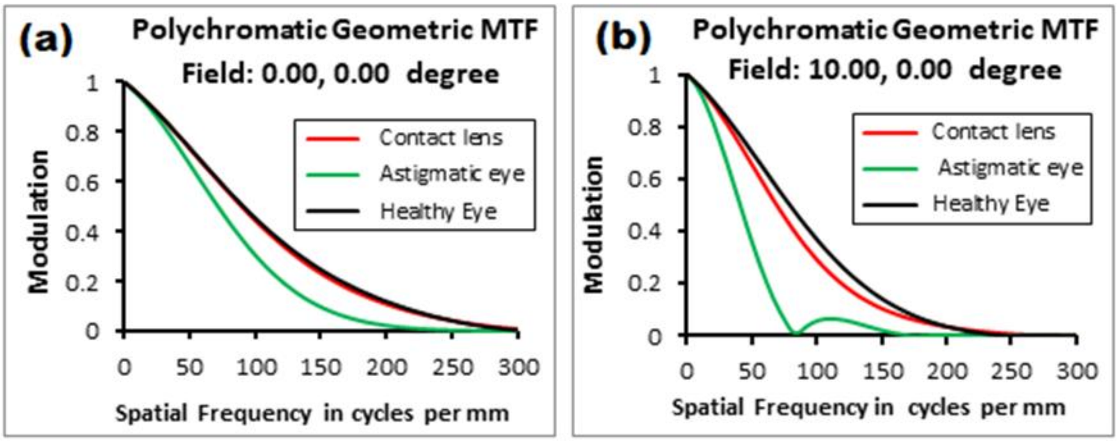


Figure 2. Diagram of the focal beam spot from distance to near (left to right) in (a) healthy eye (b) uncorrected eye (c) eye with contact lens.

Figure 3 illustrates the changes in the Modulation Transfer Function (MTF) across three different vision conditions i.e. $(0°, 10°, 20°)$. The graph presents the MTF for a healthy eye, an uncorrected eye, and eyes fitted with contact lenses. The curves demonstrate how corrective measures enhance the MTF function, leading to improved visual acuity.
Analysis of the graph shows that contact lenses provide a greater improvement in the Modulation Transfer Function (MTF) when correcting the eye's astigmatism. The MTF curves for contact lenses are higher than those of a healthy eye, indicating an enhanced capacity to resolve fine details across a range of spatial frequencies. Nonetheless, it is important to note that individual variations and specific eye conditions may affect the effectiveness of different corrective approaches.



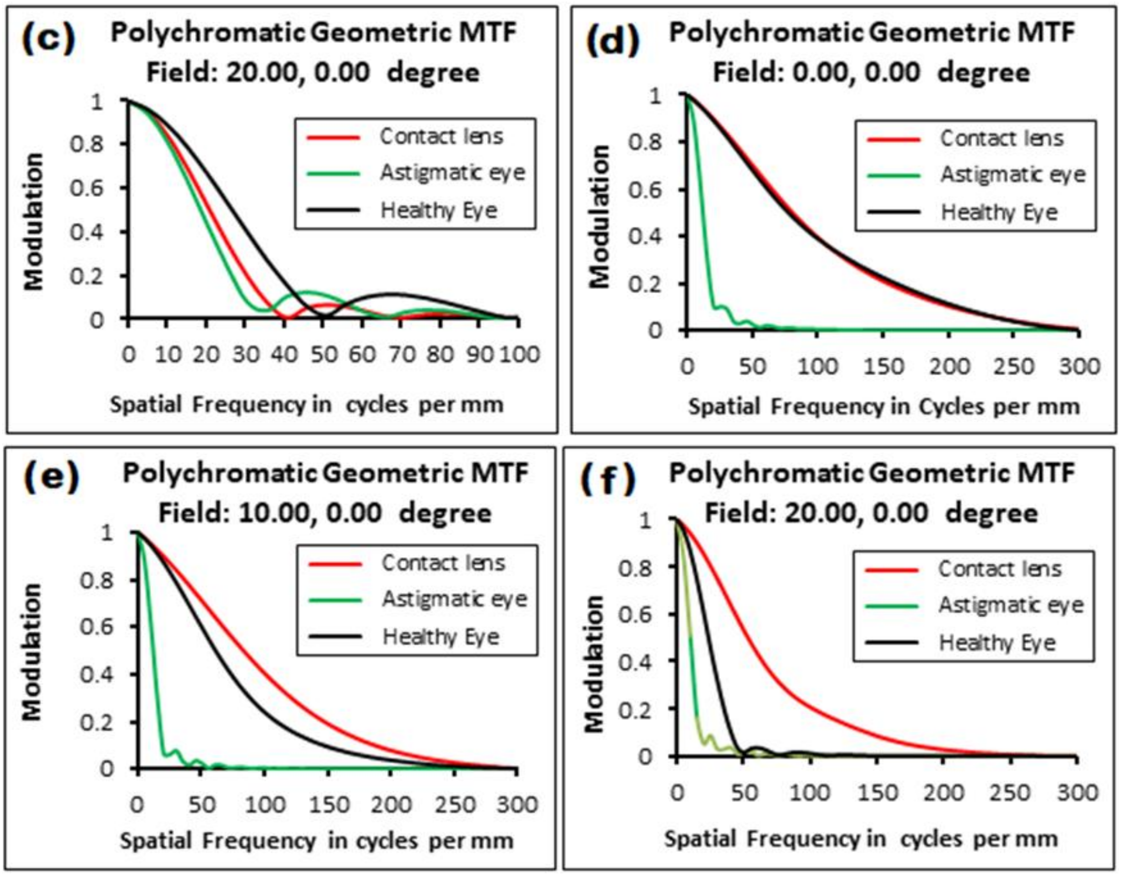


Figure 3. MTF diagrams for healthy, astigmatic, and contact lens-corrected eyes, showing sagittal- (a, b, c) and tangential- (d, e, f) modulation transfer functions at fields of view of 0°, 10°, and 20°.

Figure 3 shows the sagittal modulation transfer functions, at a 0° field of view, the lens performs identically to a healthy eye, but differences emerge at 10° and 20°, where the healthy eye shows better contrast. For the tangential modulation transfer functions, the two are also identical at 0°, but at 10° and 20°, the lens-corrected eye exhibits superior contrast to the natural eye. Based on the average MTFA of the sagittal and tangential components, we conclude that the contact lens outperforms the natural eye, providing a clearer image with less contrast loss.

The PSF is a valuable tool for analyzing the quality and resolution of the image on the retina. It describes how a point source of light is spread out on the retina due to the optical system's characteristics. By examining the PSF, we can gain insights into the sharpness and clarity of the image formed on the retina. The PSF is derived from the Fourier transform of the MTF, which allows us to analyze the different spatial frequencies present in the PSF and how they are affected by the optical system [22]. As the resolving power (MTF) increases, the size of the optical spot (PSF) decreases, indicating an improvement in image resolution. **Figure 4** compares the PSF function for eye with astigmatism before and after treatment with contact lenses. The graph shows that contact lenses are effective in improving image contrast.

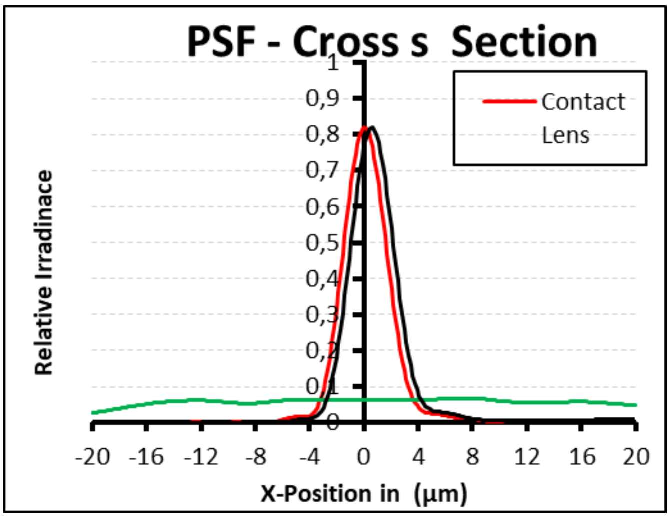


Figure 4. Point spread function (PSF) diagrams of the vision zone (0°) for a healthy eye and an astigmatic eye, prior to and following contact lens treatment.

Figure 5. Shows an image formed using the contact lens model, where polychromatic light sources produced the clearest image. In comparison, the images formed by the prepared uncorrected eye model (representing an astigmatic eye) were blurred and of poorer quality than the retinal image formed by the eye without a contact lens. The presence of spherical aberration, coma, and astigmatism prevented the formation of a sharp image, as rays from a single object point failed to converge to a common focal point. No significant improvement in image contrast was observed, which confirms the findings discussed in the MTF analysis.

Figure 5. Human eye retinal image simulation (a) healthy eye (b) uncorrected eye (c) eye with contact lens.

4. Conclusion

In this study, we employed Liu and Brennan's model using Zemax software to simulate the treatment of astigmatism with contact lenses doped with nanomaterials at a concentration of 0.01% w/v. By adjusting for the uneven curvature in the cornea & crystalline lens, these lenses are thought to be promising for optimizing astigmatism correction effectiveness. Designing and testing these contact lenses in an eye with about two diopters of astigmatism was the goal of this project, considering an aberrated eye operating under polychromatic light sources.

The simulation results analyzed through spot diagrams, Modulation Transfer Function (MTF), Point Spread Function (PSF), and retinal picture simulation—show that the PMMA-TiO_2 contact lenses may greatly improve astigmatic people's visual performance. The simulations show that retinal picture quality has improved, Spot diagrams with lower RMS values show a more homogenous and concentrated retinal picture, On the other hand, improved MTF diagrams show sharper images. A cleaner and more precise retinal image is further enhanced by the drop in aberration coefficients, which also validate a decrease is optical distortions.

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