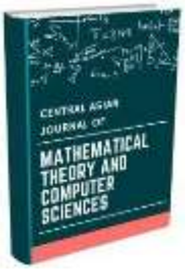

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Article

Best Proximity Point Theorem for Non-Cyclic Geraghty Contractions Mappings of Generalized Metric Space

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Abstract: The approaches for finding an idea approximate solution, known as a best proximity point, to the equation $\mathcal{F}e = e$, which is certainly unsolvable when $\mathcal{F}$ is a non-self mapping, can be determined by best proximity point theorems. This paper establishes adequate requirements for the existence of a uniqueness best (optimum) proximity point for Geraghty contractions mappings in double controlled $M_b -$ metric space. for new classes of non-self mappings known as generalized proximal contractions. Furthermore, the aforementioned best proximity point theorems can be realized as specific cases of the well-known Banach's contraction principle and more than of its expansions and modifications.

Keywords: Geraghty contractions mappings, p – property, best proximity point

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1. Introduction

In [1], Fan introduced the concept of best proximity point for non-self continuous mappings $\mathcal{F}: E \to B$, where E is a Hausdorff locally convex topological vector space B's nonempty compact convex subset. Indeed, he demonstrated that there exists x such that $\partial(e, \mathcal{F}e) = \partial(\mathcal{F}e, E)$ in metric space $\partial(B, \partial)$. In the literature, Prolla [2], Reich [3], Singh, and Sehgal [4] established several generalizations of Fan's theorem. The aforementioned definition was expanded in 2010 by S. Bacha [5] to a pair of nonempty subsets (E, D) of a metric space $\partial(B, \partial)$ in order to incorporate additional expansions of the Banach contraction principle by a best proximity theorem, assuming that B is approximately compact with respect to E .

Subsequently, a number of optimal proximity point solutions were obtained (see, for example, [6], [7], [8]. In [9], M. Jleli and B. Samet obtained the best proximity point theorems for non-self set valued mappings under the proximal orbital completeness criterion, which is less stringent than the compactness condition. The generalization of fixed point theorems is done by best proximity point theorems.

In the case of self-mappings, the optimal proximity point actually turns into a fixed point. In [10], [11], [12], some of extensions of non-self contractions for the presence of optimal proximity locations were examined. Additionally, several best proximity theorems came to be for some classes of non-self mappings in [13], [14], [15]. For a new class of non-self mappings, we examine the presence and uniqueness of optimum proximity points. We demonstrate that the findings in [16], [17] are specific instances of our primary finding.

2. Materials and Methods

Preliminaries and definitions

Definition 2.1.[3] Let B be a nonempty set and $\theta, \mu : B^2 \rightarrow [1, \infty)$ be functions. A Double controlled $M_b -$ metric on B is a function $\partial : B^2 \rightarrow [0, \infty)$ satisfies the following conditions for all $\omega, \tau, \epsilon \in B$.

(h_1) $\partial(\omega, \omega) = \partial(\tau, \tau) = \partial(\epsilon, \epsilon) \leftrightarrow \omega = \tau$

(h_2) $m_{\omega,\tau} \leq \partial(\omega, \tau)$

(h_3) $\partial(\omega, \tau) = \partial(\tau, \omega)$

(h_4) $(\partial(\omega, \tau) - m_{\omega,\tau}) \leq \theta(\omega, \epsilon)(\partial(\omega, \epsilon) - m_{\omega,\epsilon}) + \psi(\epsilon, \tau)(\partial(\epsilon, \tau) - m_{\epsilon,\tau})$

Where $m_{\omega,\tau}$ and $\mathcal{M}_{\omega,\tau}$ are defined by

1. $m_{\omega,\tau} := min\{\partial(\omega, \omega), \partial(\tau, \tau)\}$
2. $\mathcal{M}_{\omega,\tau} := max\{\partial(\omega, \omega), \partial(\tau, \tau)\}$

Then, the pair (B, ∂) is called Double controlled $M_b -$ metric space. We denoted Double controlled $M_b -$ metric space by DCM_b −MS.

Definition 2.2. Let (E, D) be a pair of nonempty subsets of DCM_b −MS. (B, ∂). we adopt the following :

1. $\partial(E, D) = \inf\{\partial(e, d) : e \in E , d \in D\}$.
2. $E_0 = \{e \in E : \partial(e, d) = \partial(E, D)\ for\ some\ d \in D\}$.
3. $D_0 = \{d \in D : \partial(e, d) = \partial(E, D)\ for\ some\ e \in E\}$.

In [8] , the authors provide adequate requirements to establish the non-emptiness of sets E_0 and D_0.

Let's interduce that the set G is the class of functions $\gamma: [0, \infty) \rightarrow [0,1]$ such that $\gamma(l_n) \rightarrow 1$ implies $l_n \rightarrow 0$ for any bounded sequence $\{l_n\}$ of positive real number.

Definition 2.3. [8] Let $\mathcal{F}: E \rightarrow B$ be a non-self mapping and (E, B) be two nonempty subsets of a DCM_b −MS. (B, ∂). If $\partial(e, \mathcal{F}e) = \partial(E, B)$, then an element $e \in E$ is called a best proximity point of $\mathcal{F}$.

According to the aforementioned notations, we have $e \in E_0$ and $\mathcal{F}e \in B_0$ If e is best proximity point for $\mathcal{F}$.

Definition 2.4.[8] Let (E, D) be a pair of nonempty subsets of DCM_b −MS. (B, ∂).with $E_0 \neq \emptyset$. Then the pair (E, D) is said to have p-property if and only if

$\partial(e_1, d_1) = \partial(E, D)\ and\ \partial(e_2, d_2) = \partial(E, D) \Rightarrow \partial(e_1, e_2) = \partial(d_1, d_2)$ for any $e_1, e_2 \in E_0$ and $d_1, d_2 \in D_0$.

<u>Remark</u> 2.5.

Let $\mathrm{E} \neq \emptyset$. Subset of $\mathrm{DCM_b}$ −MS. (B, ∂) and let $\mathrm{e_1, e_2, u_1, u_2} \in \mathrm{E}$ if

$$\partial(\mathrm{e_1, u_1}) = \partial(\mathrm{E, E}) \text{ and } \partial(\mathrm{e_2, u_2}) = \partial(\mathrm{E, E}) \Rightarrow \partial(\mathrm{e_1, e_2}) = \partial(\mathrm{u_1, u_2})$$

Then the pair $\partial(\mathrm{E, E})$ has the p – property.

Definition 2.6. [17] Let (E, D) be a pair of nonempty subsets of DCM_b −MS. (B, ∂). A mapping $\mathcal{F}: E \rightarrow D$ is said to be Geraghty – contraction if there exists $\zeta \in G$, such that for each $e_1, e_2 \in E$,

$$\partial(\mathcal{F}\, e_1, \mathcal{F}e_2) \leq \zeta(\,\partial(\,e_1, e_2))\,\partial(\,e_1, e_2).$$

<u>Remark</u> 2.7.

1. Since $\zeta : [0, \infty) \rightarrow [0,1)$, we have

$\partial(\mathcal{F}\, e_1, \mathcal{F}e_2) \leq \zeta(\,\partial(\,e_1, e_2))\,\partial(\,e_1, e_2) < \partial(\,e_1, e_2)$ for any $e_1, e_2 \in E$ with $e_1 \neq e_2$.

2. Every Geraghty – contraction is contractive mapping

Since $k \in (0,1)$, we have

$\partial(\mathcal{F}\, e_1, \mathcal{F}e_2) \leq \zeta(\,\partial(\,e_1, e_2))\partial(\,e_1, e_2)$ for each $e_1, e_2 \in E$

If $\zeta(\,\partial(\,e_1, e_2)) = k$, then

$\partial(\mathcal{F}\, e_1, \mathcal{F}e_2) \leq k\,\partial(\,e_1, e_2)$ for each $e_1, e_2 \in E$.

3. Results and Discussion

3. Principal results.

This section begins by describing our principal results.

Theorem 3.1. Let (E, D) be a pair of nonempty closed subsets of a complete DCM_b −MS. (B, ∂) such that $E_0 \neq \emptyset$. Let $\mathcal{F} : E \to D$ be Geraghty - contraction satisfying $\mathcal{F}(E_0) \subseteq D_0$. Suppose that the pair (E, D) has p-property then there exists a unique e^* such that $\partial(\mathcal{F}e^*, e^*) = \partial(E, D)$

Proof. Since $E_0 \neq \emptyset$, we take $e_0 \in E$.

As $\mathcal{F}e_0 \in \mathcal{F}(E_0) \subseteq D_0$,there exists $e_1 \in E_0$ such that $\partial(e_1, \mathcal{F}e_0) = \partial(E, D)$. Similarly, since $\mathcal{F}e_1 \in \mathcal{F}(E_0) \subseteq D_0$, there exists $e_2 \in E_0$ such that $\partial(e_2, \mathcal{F}e_1) = \partial(E, D)$. Also, there exists $e_3 \in E_0$ such that $\partial(e_3, \mathcal{F}e_2) = \partial(E, D)$

⋮

again, there exists $e_{n-1} \in E_0$ such that $\partial(e_{n-1}, \mathcal{F}e_{n-2}) = \partial(E, D)$ for $n \in N$

we have a sequence $\{e_n\} \in E_0$ satisfying $\partial(e_{n+1}, \mathcal{F}e_n) = \partial(E, D)$ for $n \in N$

since (E, D) has p – property, such that

$\partial(e_n, \mathcal{F}e_{n-1}) = \partial(E, D)$ and $\partial(e_{n+1}, \mathcal{F}e_n) = \partial(E, D) \Rightarrow \partial(e_n, e_{n+1}) = \partial(\mathcal{F}e_{n-1}, \mathcal{F}e_n)$ for any $n \in N$.

Since $\mathcal{F}$ is Geraghty – contraction, for any $n \in N$, we have that

$$\partial(e_n, e_{n+1}) = \partial(\mathcal{F}e_{n-1}, \mathcal{F}e_n) \leq \zeta(\partial(e_{n-1}, e_n))\, \partial(e_{n-1}, e_n) < \partial(e_{n-1}, e_n) \quad (4)$$

To prove that $\{e_n\}$ is Cauchy in DCM_b −MS we have to show

$\lim_{n,m\to\infty}(\partial(\sigma_n, \sigma_m) - m_{\sigma_n,\sigma_m})$and $\lim_{n,\to\infty}(\mathcal{M}_{\sigma_n,\sigma_m} - m_{\sigma_n,\sigma_m})$ exist and finite.

Suppose there exists $n_0 \in N$ such that

If $e_{n_0} = e_{n_0+1}$ this mine

$\partial(e_{n_0}, e_{n_0+1}) = \partial(e_{n_0}, e_{n_0}) = \partial(e_{n_0+1}, e_{n_0+1})$

$\partial(\mathcal{F}e_{n_0-1}, \mathcal{F}e_{n_0}) = \partial(\mathcal{F}e_{n_0-1}, \mathcal{F}e_{n_0-1}) = \partial(\mathcal{F}e_{n_0}, \mathcal{F}e_{n_0})$

$\partial(e_{n_0}, e_{n_0+1}) = \partial(\mathcal{F}e_{n_0-1}, \mathcal{F}e_{n_0})$

And consequently $\mathcal{F}e_{n_0-1} = \mathcal{F}e_{n_0}$ we have

$\partial(E, D) = \partial(e_{n_0}, \mathcal{F}e_{n_0-1}) = \partial(e_{n_0}, \mathcal{F}e_{n_0})$

And this end the prove where e_{n_0} is best proximity point.

either, If $e_{n_0} \neq e_{n_0+1}$ this mine

$\partial(e_{n_0}, e_{n_0+1}) \neq \partial(e_{n_0}, e_{n_0}) \neq \partial(e_{n_0+1}, e_{n_0+1})$

Suppose that $\partial(e_n, e_{n+1}) > 0$ for any $n \in N$.

By (2) $\{\partial(e_n, e_{n+1})\}$ is a decreasing sequence of nonnegative real numbers, and hence there exists $s \in (0,1]$ such that $\lim_{n\to\infty} \partial(e_n, e_{n+1}) = s$

We prove $s = 0$

Suppose $0 < s < 1$ then from (4) we have

$$\partial(e_n, e_{n+1}) = \partial(\mathcal{F}e_{n-1}, \mathcal{F}e_n) \leq \zeta(\partial(e_{n-1}, e_n))\, \partial(e_{n-1}, e_n)$$

$0 < \frac{\partial(e_n, e_{n+1})}{\partial(e_{n-1}, e_n)} < \zeta(\partial(e_{n-1}, e_n)) < 1$ for any $n \in N$.

Implies that $\lim_{n\to\infty} \zeta\partial(e_{n-1}, e_n) = 1$ and since $\zeta \in G$, We obtain $s = 0$ and this contraction to assumption we have $\lim_{n\to\infty} \partial(e_n, e_{n+1}) = 0$.

We note that since $\partial(e_{n+1}, \mathcal{F}e_n) = \partial(E, D)$ for any $n \in N$, for $i, j \in N$, we get $\partial(e_i, \mathcal{F}e_{i-1}) = \partial(e_j, \mathcal{F}e_{j-1}) = \partial(E, D)$ and since (E, D) satisfying p -property, we have

$$\partial(e_i, e_j) = \partial(\mathcal{F}e_{i-1}, \mathcal{F}e_{j-1})$$

Now, we prove that $\{e_n\}$ is Cauchy sequence.

In the contrary case, we have that $\lim_{m,n\to\infty} \sup(\partial(e_n, e_m) - m_{e_n,e_m}) > 0$

$$\begin{aligned}\partial(e_n, e_m) - m_{e_n,e_m} &\\ &\leq \theta(e_n, e_{n+1})(\partial(e_n, e_{n+1}) - m_{e_n,e_{n+1}}) \\ &+ \psi(e_{n+1}, e_m)(\partial(e_{n+1}, e_m) - m_{e_{n+1},e_m})\end{aligned}$$

Let $l_1 = \theta(e_n, e_{n+1})(\partial(e_n, e_{n+1}) - m_{e_n,e_{n+1}})$

$$(\partial(e_n, e_m) - m_{e_n,e_m}) \leq l_1 + \psi(e_{n+1}, e_m)\left\{\begin{matrix}\theta(e_{n+1}, e_{m+1})(\partial(e_{n+1}, e_{m+1}) - m_{e_{n+1},e_{m+1}}) \\ +\psi(e_{m+1}, e_m)(\partial(e_{m+1}, e_m) - m_{e_{m+1},e_m})\end{matrix}\right\}$$

$$\left(\partial(e_n,e_m)-m_{e_n,e_m}\right)\le l_1+\psi(e_{n+1},e_m)\theta(e_{n+1},e_{m+1})\left(\partial(e_{n+1},e_{m+1})-m_{e_{n+1},e_{m+1}}\right)$$
$$+\psi(e_{n+1},e_m)\psi(e_{m+1},e_m)\left(\partial(e_{m+1},e_m)-m_{e_{m+1},e_m}\right)$$

Let $l_2=\psi(e_{n+1},e_m)\psi(e_{m+1},e_m)\left(\partial(e_{m+1},e_m)-m_{e_{m+1},e_m}\right)$

And by equation (3) and since $\partial(e_{n+1},e_{m+1})=\partial(\mathcal{F}e_n,\mathcal{F}e_m)$ we have

$$\left(\partial(e_n,e_m)-m_{e_n,e_m}\right)\le l_1+\psi(e_{n+1},e_m)\theta(e_{n+1},e_{m+1})\left(\partial(\mathcal{F}e_n,\mathcal{F}e_m)-m_{e_{n+1},e_{m+1}}\right)+l_2$$

$$\left(\partial(e_n,e_m)-m_{e_n,e_m}\right)$$
$$\le l_1+\psi(e_{n+1},e_m)\theta(e_{n+1},e_{m+1})\left(\zeta\left(\partial(e_n,e_m)\right)\partial(e_n,e_m)-m_{e_{n+1},e_{m+1}}\right)+l_2$$

$$\partial(e_n,e_m)=\partial(\mathcal{F}e_{n-1},\mathcal{F}e_{m-1})\le\zeta\left(\partial(e_{n-1},e_{m-1})\right)\partial(e_{n-1},e_{m-1})$$

$0<\frac{\partial(e_n,e_m)}{\partial(e_{n-1},e_{m-1})}<\zeta\left(\partial(e_{n-1},e_{m-1})\right)<1$ for any $n\in N$.

Implies that $\lim_{n\to\infty}\zeta\partial(e_{n-1},e_{m-1})=1$ and since $\zeta\in G$, we have $\lim_{n\to\infty}\partial(e_n,e_m)=0$.

$$\mathcal{M}_{e_n,e_m}=max\{\partial(e_n,e_n),\partial(e_m,e_m)\}=\partial(e_n,e_n)$$

Hence, we get

$$\begin{aligned}\mathcal{M}_{e_n,e_m}-m_{e_n,e_m}&\le\mathcal{M}_{e_n,e_m}\\&=\partial(e_n,e_n)\\&\le\zeta\left(\partial(e_{n-1},e_{n-1})\right)\partial(e_{n-1},e_{n-1})\\&\vdots\\&\le\zeta^n\prod_{i=0}^{n-1}\left(\partial(e_0,e_0)\right)\partial(e_0,e_0)\end{aligned}$$

Letting $n\to\infty$, we deduce that

$$\lim_{n,m\to\infty}\left(\mathcal{M}_{e_n,e_m}-m_{e_n,e_m}\right)=0$$

We conclude that $\{e_n\}$ is $\partial-$ Cauchy in E. Since E *is* $\partial-$ complete, $\{e_n\}$ converges to a point $e\in E$ so that we have

$$\lim_{n,m\to\infty}\left(\partial(e_n,e)-m_{e_n,e}\right)=0$$

Since $\{e_n\}\subset E$ and E is closed subset of DCM_b −MS, we can fined $e^*\in E$ such that $e_n\to e^*$

Since any Geraghty – contraction is contractive mapping and hence continuous, we have $\mathcal{F}e_n\to\mathcal{F}e^*$

This implies that $\partial(e_{n+1},\mathcal{F}e_n)\to\partial(e^*,\mathcal{F}e^*)$

If we take the sequence $\{\partial(e_{n+1},\mathcal{F}e_n)\}=\partial(E,D)$, we get $\partial(e^*,\mathcal{F}e^*)=\partial(E,D)$

e^* is best proximity point of $\mathcal{F}$.

To prove e^* is unique, suppose e_1,e_2 are two (bpp) of $\mathcal{F}$ with $e_1\ne e_2$

i.e. $\partial(e_1,\mathcal{F}e_1)=\partial(E,D)$ and $\partial(e_2,\mathcal{F}e_2)=\partial(E,D)$

Since (E,D) has p- property, we have $\partial(e_1,e_2)=\partial(\mathcal{F}e_1,\mathcal{F}e_2)$

Since $\mathcal{F}$ is Geraghty – contraction, we have

$$\partial(e_1,e_2)=\partial(\mathcal{F}e_1,\mathcal{F}e_2)\le\zeta\left(\partial(e_1,e_2)\right)\partial(e_1,e_2)<\partial(e_1,e_2)$$

Which is a contradiction.

Therefore, $e_1=e_2$.

Example 3.2. consider $B=R^2$ and let $\theta,\psi:B^2\to[1,\infty)$ be functions, a DCM_b −M $\partial:R^2\to R^+$ defined by $\partial(e_1,e_2)=|e_1-e_2|$ for all $e_1,e_2\in R$ and $\theta(e_1,e_2)=\psi(e_2,e_3)=(1,1)$, Let E and D be closed subsets of B which defined by $E=\{0\}\times R^+$, $D=\{1\}\times R^+$. and $\partial(E,D)=1$.Moreover, suppose $E_0=E$ and $D_0=D$. Assume $\mathcal{F}:E\to D$ be the mapping defined by $\mathcal{F}(0,e)=(1,\ln(1+e))$ for any $(0,e)\in E$ we show that $\mathcal{F}$ is Geraghty – contraction.

Prove: let $(0,e_1),(0,e_2)\in E$ with $e_1\ne e_2$, we have

$$\begin{aligned}\partial\left(\mathcal{F}(0,e_1),\mathcal{F}(0,e_2)\right)&=\partial\left((1,\ln(1+e_1)),(1,\ln(1+e_2))\right)\\&=|\ln(1+e_1)-\ln(1+e_2)|\\&=\left|\ln\left(\frac{1+e_1}{1+e_2}\right)\right|\end{aligned}\qquad(2.2.1)$$

Now to prove that

$$\left|\ln\left(\frac{1+e_1}{1+e_2}\right)\right|\le\ln(1+|e_1-e_2|)\ .\qquad(2.2.2)$$

Suppose that $e_1>e_2$, the same prove when $e_1<e_2$.

Then, since $\gamma(x)=\ln(1+x)$ is strictly increasing in $[0,\infty)$, we have

$$\ln\left(\frac{1+e_1}{1+e_2}\right)=\ln\left(\frac{1+e_2+e_1-e_2}{1+e_2}\right)$$
$$=\ln\left(1+\frac{e_1-e_2}{1+e_2}\right)\le \ln(1+e_1-e_2\,)=\ln(1+|e_1-e_2|\,)\,.$$
$$\partial\big(\mathcal{F}(0,e_1),\mathcal{F}(0,e_2)\big)=\left|\ln\left(\frac{1+e_1}{1+e_2}\right)\right|\le \ln(1+|e_1-e_2|)$$
$$=\frac{\ln(1+|e_1-e_2|)}{|e_1-e_2|}|e_1-e_2|$$
$$=\frac{\gamma\Big(\partial\big((0,e_1),(0,e_2)\big)\Big)}{\partial\big((0,e_1),(0,e_2)\big)}.\partial\big((0,e_1),(0,e_2)\big)\,.$$
$$=$$
$$\zeta\Big(\partial\big((0,e_1),(0,e_2)\big)\Big).\partial\big((0,e_1),(0,e_2)\big), \qquad (2.2.3)$$

Where $\gamma(x)=\ln(1+x)$ for $x\ge 0$, and $\zeta=\frac{\gamma(x)}{x}$ for $x>0$ and $\zeta(0)=0$.

Obviously, when $e_1=e_2$, the inequality (2.10.3) is hold.

$\zeta(x)=\frac{\ln(1+x)}{x}\in\mathcal{F}$ by using elemental calculus.

Since $\mathcal{F}$ is a Geraghty – contraction.

Since (E,D) hold p – property. And, if

$$\partial\big((0,e_1),(1,c_1)\big)=\sqrt{1+(e_1-c_1)^2}=\partial(E,D)=1,$$
$$\partial\big((0,e_2),(1,c_2)\big)=\sqrt{1+(e_2-c_2)^2}=\partial(E,D)=1,?$$

then $e_1=c_1$ and $e_2=c_2$.

$$\partial\big((0,e_1),(0,e_1)\big)=|e_1-e_2|=|c_1-c_2|=\partial\big((1,c_1),(1,c_2)\big)$$

By theorem 2,9, $\mathcal{F}$ has a unique best proximity point this point is $(0,0)\in E$.

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