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# VIBRATIONS OF AN INFINITE PIECE-HOMOGENEOUS TWO-LAYER PLATE UNDER THE INFLUENCE OF NORMAL LOAD

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**Abstract:** In given article it is considered influence of normal loading on an infinite kusochno-homogeneous two-layer plate, when materials top and нижного plate layers elastic. It is defined cross-section displacement of points of a plane of contact of the two-layer plate, satisfying to the approached equation received in work [1], replacing only viscoelastic operators with elastic factors of Ljame accordingly.

For a rectangular infinite two-layer kusochno-homogeneous plate at nonzero initial conditions, frequencies of own fluctuations are calculated, and the analytical decision of this problem is under construction.

The received theoretical results for the decision of dynamic problems of cross-section fluctuation of kusochno-homogeneous two-layer plates of a constant thickness taking into account elastic properties of their material allow to count more precisely cross-section displacement of points of a plane of contact of plates at normal external loadings.

**Keywords:** the fluctuation equations, a two-layer plate, displacement, elastic, viscoelastic, edge conditions, initial conditions, the operator, factors of Ljame, the differential equation, integral of Fure, complex frequency.

## Introduction

In real structures, the destruction of their elements is usually accompanied by impact loads.

In this work, a solution is constructed on the vibrations of an infinite two-layer plate under the action of a normal load applied to the surface of a two-layer plate.

The problem is reduced to solving an approximate equation for the transverse displacement W of points of the contact plane of a two-layer plate of constant thickness, obtained in [1] and [2].

$$Q_1\left(\frac{\partial^4 W}{\partial t^4}\right)+Q_2\left(\Delta\frac{\partial^2 W}{\partial t^4}\right)+Q_3(\Delta^2 W)+Q_4\left(\frac{\partial^6 W}{\partial t^6}\right)+$$
$$+Q_5\left(\Delta\frac{\partial^4 W}{\partial t^4}\right)+Q_6\left(\Delta^2\frac{\partial^2 W}{\partial t^2}\right)+Q_7(\Delta^3 W)=F(x,y,t) \qquad (1)$$

,

where the coefficients Q_j are determined by the formula obtained in [2].

Assuming the load F(x,y,t) to be even in (x, y), the transverse displacement W will be sought in the form of the Fourier integrals

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$$W=\int_0^\infty\int_0^\infty W_0\cos(kx)\cos(qy)dkdq \tag{2}$$

Substituting (2) into equations (1), for W_0 we obtain the ordinary differential equation

$$W_0^{VI}+A_1W_0^{IV}+A_2W_0^{II}+A_3W_0=F_0(k,q,t), \tag{3}$$

where the coefficients $A_j$ and $F_0(k,q,t)$ are equal:

$$A_1=\frac{Q_1'-\gamma^2Q_5'}{Q_4'};\ \ A_2=\frac{\gamma^2(Q_2'-\gamma^2Q_6')}{Q_4'};\ \ A_3=\frac{\gamma^4(Q_3'-\gamma^2Q_7')}{Q_4'}$$

$$F_0(k,q,t)=\int_0^\infty\int_0^\infty F(x,y,t)\cos(kx)\cos(qy)dxdy,$$

and the coefficients $Q_j'$ are determined by the formulas

$$Q_1'=P_2^2(1+h\rho)^2;$$

$$Q_2'=-2P_2^2(2(P_2D_0+hD_1)(1+h\rho)+$$
$$+(P_2-1)((1+h)-(D_0+hD_1\rho)));$$

$$Q_3'=4(P_2-1)(P_2D_0+h^2D_1+2hP_2D_0);$$

(4)

$$Q_4'=-\frac{1}{6}P_2^2((3h^2\rho^2+(1+4h\rho))(2-D_0)+$$
$$+h^2P_2(3+h\rho(h\rho+4))(2-D_1));$$

$$Q_5'=-\frac{1}{6}P_2\left((2P_2(4D_0(1-D_0)+1)+(P_2-1)(4-D_0^2)\right)-$$
$$-P_2h^2\rho^2(2(4D_1^2-4D_1-1)-(P_2-1)D_1(2-D_1))+$$
$$+6h^2(\rho(4(P_2^2D_0+D_1)+(P_2-1)(2P_2(1-D_0)-P_2D_1(2-D_0)+$$
$$+D_1(1+D_0)))+P_2(1+\rho^2))+\ +2h(2P_2\rho(2+4D_0-D_0^2)-$$
$$-h^2(2P_2-P_2D_1+5D_1-D_1^2))+(P_2-1)(4-3D_0)+ \tag{4}$$
$$+2D_1(4-D_0))+2P_2h\rho^2D_0(4-D_1));$$

$$Q_6'=\frac{1}{3}P_2(2D_0(3P_2-4D_0-1)+(P_2-1)(2+9D_0-3D_0^2))+$$

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$$+h_1^4 P_2 \rho(4D_1(1-2D_1)-4D_1+(P_2-1)D_1(3-D_1))+$$

$$+3h^2(4P_2D_0(P_2(1-D_1)-D_1)-(P_2-1)(2(P_2-1)D_1(1-D_0)-$$

$$-P_2(2-D_0-4D_0D_1))+P_2\rho(4D_1(1+D_0+P_2D_0)-$$

$$-(P_2-1)(6D_0D_1(P_2-1)-6P_2D_0+D_1)))+$$

$$+2hP_2(2(2D_1(1+2D_0)++(P_2-1)(1+2D_0-D_0^2))+$$

$$+h_1^2(2(P_2-1)+D_1(P_2+3))-$$

$$-4P_2\rho D_0(1+{h_0}^2(2(P_2-1)(1-D_1)+P_2D_1+(1+D_1)))));$$

$$Q_7' = \frac{2}{3}(P_2D_0(4D_0-5(P_2-1)+h_1^4D_1\left(4D_1-(P_2-1)\right)-$$

$$-3h^2\left(8P_2D_0D_1+(P_2-1)\left(3P_2D_0-(2P_2+1)D_0D_1-D_1(1-D_1)\right)\right)-$$

$$-4hP_2D_0\left(2D_1+(P_2-1)\right)+h_1^2(2(P_2-1)+(P_2+1)D_1)));$$

and γ is determined by the formula

$$\gamma^2 = h_0^2(k^2+q),$$

and immeasurable parameters were introduced:

$$h=\frac{h_1}{h_0};\ \rho=\frac{\rho_1}{\rho_0};\ b=\frac{b_0}{b_1};\ P_2=\frac{\mu_0}{\mu_1};\ D_0=\frac{1}{2(1-\nu_0)};\ D_1=\frac{1}{2(1-\nu_1)}.$$

For $\xi$ from equation (3) we obtain the frequency equation

$$\xi^6+A_1\xi^4+A_2\xi^2+A_3=0 \quad (5)$$

frequency equation (5) has purely imaginary roots, i.e., frequencies of own fluctuations.

Then, the common decision of the homogeneous differential equation (4) is equal

$$W_{og}=C_1\cos(\xi_1 t)+C_2\sin(\xi_1 t)+C_3\cos(\xi_2 t)+$$
$$+C_4\sin(\xi_2 t)+C_5\cos(\xi_3 t)+C_6\sin(\xi_3 t). \quad (6)$$

Applying a method of a variation of any constants, for $C_j'$ we will receive:

$$C_1' = \frac{1}{\xi_1(\xi_1^2-\xi_2^2)(\xi_1^2-\xi_3^2)} F_0 \, sin(\xi_1 t);$$
$$C_2' = -\frac{1}{\xi_1(\xi_1^2-\xi_2^2)(\xi_1^2-\xi_3^2)} F_0 \, cos(\xi_1 t);$$
$$C_3' = -\frac{1}{\xi_2(\xi_1^2-\xi_2^2)(\xi_1^2-\xi_3^2)} F_0 \, sin(\xi_2 t); \qquad (7)$$
$$C_4' = \frac{1}{\xi_2(\xi_1^2-\xi_2^2)(\xi_1^2-\xi_3^2)} F_0 \, cos(\xi_2 t);$$
$$C_5' = \frac{1}{\xi_3(\xi_2^2-\xi_3^2)(\xi_1^2-\xi_3^2)} F_0 \, sin(\xi_3 t);$$
$$C_6' = -\frac{1}{\xi_3(\xi_2^2-\xi_3^2)(\xi_1^2-\xi_3^2)} F_0 \, cos(\xi_3 t).$$

private decision of the differential equation (3) we will write down in a kind

$$W = \frac{1}{(\xi_1^2-\xi_2^2)(\xi_2^2-\xi_3^2)(\xi_3^2-\xi_1^2)} \{ \frac{\xi_2^2-\xi_3^2}{\xi_1} \int_0^t F_0(k,q,\zeta) \, sin[\xi_1(t-\zeta)] d\zeta + \\ + \frac{\xi_3^2-\xi_1^2}{\xi_2} \int_0^t F_0(k,q,\zeta) \, sin[\xi_2(t-\zeta)] d\zeta + \\ + \frac{\xi_1^2-\xi_2^2}{\xi_3} \int_0^t F_0(k,q,\zeta) \, sin[\xi_3(t-\zeta)] d\zeta \}. \qquad (8)$$

Satisfying with a zero initial condition, i.e.,

$$W_0 = \frac{\partial W_0}{\partial t} = \frac{\partial^2 W_0}{dt^2} = \ldots = \frac{\partial^5 W_0}{\partial t^5} = 0, \qquad (9)$$

We find that $C_1' = C_2' = \ldots = C_6' = 0$. Then, the decision of a problem for displacement $W$ looks like

$$W = \int_0^\infty \int_0^\infty \frac{cos(kx) \, cos(qy)}{(\xi_1^2-\xi_2^2)(\xi_2^2-\xi_3^2)(\xi_3^2-\xi_1^2)} \{ \frac{\xi_2^2-\xi_3^2}{\xi_1} \times \\ \times \int_0^t F_0(k,q,\zeta) \, sin[\xi_1(t-\zeta)] d\zeta + \\ + \frac{\xi_3^2-\xi_1^2}{\xi_2} \int_0^t F_0(k,q,\zeta) \, sin[\xi_2(t-\zeta)] d\zeta + \\ + \frac{\xi_1^2-\xi_2^2}{\xi_3} \int_0^t F_0(k,q,\zeta) \, sin[\xi_3(t-\zeta)] d\zeta \} dkdq \qquad (10)$$

Let, if

$$F(x,y,t) = \sigma_0 \delta(x) \delta(y) \delta(z),$$

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Where – a $\sigma_0$ constant of dimension of pressure;

$\delta(\zeta)$ - delta - function of Diraka.

Then, problem decisions will register in a kind

$$W=\sigma_0\int_0^\infty\int_0^\infty\frac{cos(kx)\,cos(qy)}{(\xi_1^2-\xi_2^2)(\xi_2^2-\xi_3^2)(\xi_3^2-\xi_1^2)}[\frac{\xi_2^2-\xi_3^2}{\xi_1}sin(\xi_1 t)+ +\frac{\xi_3^2-\xi_1^2}{\xi_2}sin(\xi_2 t)+\frac{\xi_1^2-\xi_2^2}{\xi_3}+sin(\xi_3 t)]dkdq \tag{11}$$

**Conclusions**

From the analytical decision of a problem on influence of normal loading on a surface of a two-layer plate follows that the deflection depends on geometrical and mechanical characteristics of a material of a plate, and also allows to describe precisely enough tensely - the deformed status of a plate in its any point eventually.

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