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## SYSTEMS OF BATTACHARY LOWER BOUNDS AND SYMPTOMS OF EQUILIBRIUM IN THEM

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**Abstract:** This article explores the correlation coefficient, a vital concept in mathematical statistics. It discusses correlation and regression concepts, their interrelation, and estimation methods. Various correlation coefficients like Pearson, Spearman, and Kendall are detailed, along with their unique properties and applications.

**Key words:** Correlation coefficient, covariance, regression, statistical analysis, Pearson, Spearman, Kendall.

### Introduction.

It is known from mathematical statistics that an infinite number of experiments observing the random variable  $X=X(\omega)$  defined in the probability space  $(\Omega, \mathcal{A}, P)$  are conducted independently under the same conditions, and their results are independent. and  $\{X_1, X_2, \dots\}$  is considered to be a sequence of uniformly distributed random variables, and in  $n$  experiments  $\{X_1, X_2, \dots, X_n\}$  is a statistical set, that is, a statistical sample is observed. For each fixed  $\omega$ , the actual value of this selection consists of the numbers  $\{X_1, X_2, \dots, X_n\}$ , respectively. We define a statistical sample and its values as vectors  $X(n)=\{X_1, X_2, \dots, X_n\}$  and  $x(n)=\{x_1, x_2, \dots, x_n\}$ , respectively. We denote the set of all possible values of  $x(n)$  by  $\mathfrak{X}$ . This set is called the selected set. Let  $\mathcal{B}$  be the Borel  $\sigma$ -algebra composed of subsets of  $\mathfrak{X}$ . In the  $(\mathfrak{X}, \mathcal{B})$  dimensional space, the selection  $X(n)$  produces this distribution  $P(\mathcal{B})=P(\omega: X(n) \in \mathcal{B}), \mathcal{B} \in \mathcal{B}$ . The main task of mathematical statistics is to determine or evaluate the distribution  $P$  through the results of sampling  $X(n)$  [4]. Thus, the distribution  $P$  belongs to a family  $\{P\}$ .

This  $(\mathfrak{X}, \mathcal{B}, \{P\})$  family of probability spaces is called a statistical model. If the family  $\{P\}$  is parameterized, i.e. depends on some parameter  $\theta$ , then the  $(\mathfrak{X}, \mathcal{B}, \{P_\theta, \theta \in \Theta\})$  model is called a parametric statistical model.

The main requirement for point estimates is that they have the properties of reasonableness, non-displacement, and minimality of the risk function. The property of validity is manifested in the fact that the sample size is large enough. But when the sample size is limited, the minimum property of the risk function is studied. For unshifted statistical estimates, the risk function overlaps with the variance, being quadratic. In this case, the estimate with the smallest variance is called the best, effective estimate.

A lower bound for the variance can be specified when the family of distributions satisfies certain conditions. These conditions are called Kramer-Rao regularity conditions [1-3]. Swedish mathematician Harald Kramer and Indian mathematician Kaliampudi Rao independently introduced these conditions when proving the inequality, so the Kramer-Rao inequality is named after them.  $(\mathfrak{X}, \mathfrak{B}, P\Theta)$  let  $X(n)$  be a sample of size  $n$  and its values be  $x(n)$  in the statistical model. We define this  $L(\theta, x) = L(\theta, x_1, x_2, \dots, x_n)$  function of similarity to the truth and assume that the following regularity conditions are valid.

- i)  $\theta$  everywhere differentiable with respect to  $L > 0$  and  $\theta$ ;
- ii) Let this function  $U(\theta, x) = \frac{\partial \ln L(\theta, x)}{\partial \theta}$  have finite variance;
- iii) Let the following equality hold for the statistic  $\hat{\theta}_n(x)$  with a finite second-order moment.

$$\frac{\partial}{\partial \theta} \int \hat{\theta}_n(x) L(\theta, x) dx = \int \hat{\theta}_n(x) \frac{\partial}{\partial x} L(\theta, x) dx. \quad (1)$$

If  $\hat{\theta}_n(x)$  satisfying the above conditions is an unshifted estimate for the differentiable function  $\tau(\theta)$ , then the following inequality holds for them.

$$D_{\theta}(\hat{\theta}_n(x)) \geq \frac{(\tau'(\theta))^2}{nI(\theta)}, \quad (2)$$

where  $I(\theta) = M\left(\frac{dx \ln L(\theta, x)}{d\theta}\right)^2$  –Fisher information,  $L(\theta, x)$  is a density function, if  $X$  is continuous if  $\{X=t\}$  is the probability of the event if and only if the following inequality holds

$$\frac{d \ln L(\theta, x)}{d\theta} = a(\theta)(\hat{\theta}_n(x) - \tau(\theta)).$$

The following special case of the Kramer-Rao inequality is also used. When the regularity conditions are met, if the  $\theta(x)$ -estimate is an unshifted estimate for the  $\theta$  parameter, then

$$D_{\theta}(\hat{\theta}_n(x)) \geq \frac{1}{I_n(\theta)},$$

inequality is appropriate. In this case,  $\hat{\theta}_n(x) - \theta = a(\theta) \cdot U(\theta, x)$  is valid only.

If equality holds in inequality (2), then the estimate  $\hat{\theta}_n(x)$  is called an effective estimate in the Kramer-Rao sense. As the size of the inequality  $n$  increases, the asymptotic efficiency of estimates can be calculated as follows:

$$0 \leq \text{eff}(\hat{\theta}_n, \theta) = \frac{1}{I_n(\theta) D_{\theta}(\hat{\theta}_n)} \leq 1. \quad (4).$$

In this paper, we discuss and investigate Bhattacharya's system of lower bounds and related statistical issues [2,4].

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