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FRACTION IN ORDER SIMPLE DIFFERENTIAL EQUATIONS. IN CAPUTO'S SENSE FRACTION IN ORDER DIFFERENTIAL EQUATIONS FOR KOSHI ISSUE

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Annotation: Today's in the day differential equations and mathematics and physics equations directions wide spreading from directions one this fraction in order derivative and fraction are integral equations of order . Our life during many fields basically physics , chemistry , biology and etc in the fields processes fraction in order equations with expressing them _ in learning to us fraction in order equations help will give . In life many in processes them manage important importance occupation is enough For example , heat spread in the process something from the border the heat management

Keywords: Fraction in order derivative , Caputo , Cauchy problem , Volterra integral , Mittag-Leffler function , one sexual

Fraction in order simple differential equations . In Caputo's sense fraction in order differential equations for Koshi issue

This In the section we mean Caputo linear fraction in order differential of Eqs sure solutions let's make In this : ${}^{c}D_{a+}^{\alpha}(\mathrm{y})(x)$ if $\alpha > 0$

when $C_{\gamma}^{\alpha,n-1}[a,b](n=[a]+1),$ in space defined Caputo fraction in order derivative .

First we $\alpha > 0$ when initial conditions with given Koshi the issue seeing let's go :

$$({}^{c}D_{a+}^{\alpha}\,\mathrm{y})(x) - \lambda y(x) = f(x) \quad (a \le x \le b;\; n-1 < \alpha < n;\; n \in \sim;\lambda \in \sim),$$

(2.1)

$$y^{(k)}(a) = b_k, \quad (b_k \in \sim; k = 0,\dots,n-1). \tag{2.2}$$

Hypothesis let's do $f(x) \in C_{\gamma}[a,b] \quad (0 \le \gamma < 1, \gamma \le \alpha)$ let it be In that case $C^{n-1}[a,b]$in space (2.1), (2.2) Cauchy issue the following to the Volterra integral equation equivalent equation will be :

$$y(x) = \sum_{j=0}^{n-1} \frac{b_j}{j!}(x-a)^j + \frac{\lambda}{\Gamma(\alpha)} \int_a^x \frac{y(t)}{(x-t)^{1-\alpha}} dt + \frac{1}{\Gamma(\alpha)} \int_a^x \frac{f(t)}{(x-t)^{1-\alpha}} dt \quad (2.2.3)$$

We have this integral equation consecutively approach method through the solution we find To this method according to as follows sequence dry we get :

$$y_0(x)=\sum_{j=0}^{n-1}\frac{b_j}{j!}(x-a)^j \tag{2.4}$$

$$y_m(x)=y_0(x)+\frac{\lambda}{\Gamma(\alpha)}\int_a^x\frac{y_{m-1}(t)}{(x-t)^{1-\alpha}}dt+\frac{1}{\Gamma(\alpha)}\int_a^x\frac{f(t)}{(x-t)^{1-\alpha}}dt \quad (m\in N) \tag{2.5}$$

These approximations (2.4) and the following

$$\left(\mathrm{I}_{a+}^{\alpha}f\right)(x)=\frac{1}{\Gamma(\alpha)}\int_a^x\frac{f(t)}{(x-t)^{1-\alpha}}dt \quad (x>a;\ \Re(a)>0) \tag{*}$$

from equality using , in the form of an operator writing we get can :

$$y_m(x)=y_0(x)+\lambda\left(I^{\alpha}y_{m-1}\right)(x)+\left(I^{\alpha}f\right)(x) \tag{2.6}$$

By the formula above $y_1(x)$ the counting we can

$$y_1(x)=y_0(x)+\lambda\left(I^{\alpha}y_0\right)(x)+\left(I^{\alpha}f\right)(x)=$$

$$=\sum_{j=0}^{n-1}b_j\sum_{k=0}^{1}\frac{\lambda^k(x-a)^{\alpha k+j}}{\Gamma(\alpha k+j+1)}+\frac{1}{\Gamma(\alpha)}\int_a^x(x-t)^{\alpha-1}f(t)dt$$

To the above similar $y_2(x)$ also for the following the formula writing we get :

$$y_2(x)=y_0(x)+\lambda\left(I^{\alpha}y_1\right)(x)+\left(I^{\alpha}f\right)(x)$$

From this ,

$$y_2(x)=y_0(x)+\lambda\left(I^{\alpha}y_1\right)(x)+\left(I^{\alpha}f\right)(x)$$

This as follows concisely to write can :

$$y_2(x)=\sum_{j=0}^{n-1}b_j\sum_{k=0}^{2}\frac{\lambda^k(x-a)^{\alpha k+j}}{\Gamma(\alpha k+j+1)}+\int_a^x\left[\sum_{k=1}^{2}\frac{\lambda^{k-1}}{\Gamma(\alpha k)}(x-t)^{\alpha k-1}\right]f(t)dt \tag{2.7}$$

The process continue bringing the following sequence harvest we do :

$$y_m(x)=\sum_{j=0}^{n-1}b_j\sum_{k=0}^{m}\frac{\lambda^k(x-a)^{\alpha k+j}}{\Gamma(\alpha k+j+1)}+\int_a^x\left[\sum_{k=1}^{m}\frac{\lambda^{k-1}}{\Gamma(\alpha k)}(x-t)^{\alpha k-1}\right]f(t)dt \tag{2.8}$$

m in this sequence to infinity yearning to the limit if we pass (2.3) of the integral equation to the solution we will come

$$y(x)=\sum_{j=0}^{n-1}b_j\sum_{k=0}^{\infty}\frac{\lambda^k(x-a)^{\alpha k+j}}{\Gamma(\alpha k+j+1)}+\int_a^x\left[\sum_{k=1}^{\infty}\frac{\lambda^{k-1}}{\Gamma(\alpha k)}(x-t)^{\alpha k-1}\right]f(t)dt \tag{2.9}$$

Now this the solution appearance condense for citation in chapter 1 passed

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)} \tag{2.11}$$

Mittag-Leffler from the function we use Necessary in places this function expression if we replace it , the following in appearance to the solution have we will be :

$$y(x)=\sum_{j=0}^{n-1} b_j (x-a)^j E_{\alpha,j+1}\left[\lambda(x-a)^{\alpha}\right]+\int_a^x (x-t)^{\alpha-1}E_{\alpha,\alpha}\left[\lambda(x-t)^{\alpha}\right]f(t)dt \quad (2.10)$$

This function (2.3) of Volterra's integral equation the solution will be and therefore , (2.1), (2.2) Cauchy the solution of the problem represents _

Example 2.1. Caputo's meaning to us Koshi type problem is given be :

$$({}^cD^{\alpha}\,\mathrm{y})(x)-\lambda y(x)=f(x), \quad y(+0)=b \ \ (b\in \sim) \qquad (2.11)$$

$0<\alpha<1$ va $\lambda\in\sim$ when find the solution .

$$I^{\alpha}({}^cD^{\alpha}\,\mathrm{y})(x)=I^{\alpha}\lambda y(x)+I^{\alpha}f(x)$$

$$y(x)-y(+0)=I^{\alpha}\lambda y(x)+I^{\alpha}f(x)$$

$$y(x)=y(+0)+I^{\alpha}\lambda y(x)+I^{\alpha}f(x)$$

Integral equation solve for don't go go away approach method we use

$$y_m(x)=b+\lambda I^{\alpha}y_{m-1}(x)+I^{\alpha}f(x) \quad (2.12)$$

By the formula above $y_1(x)$ the counting we can and the first approach as $y_0(x)=b$ from we use

$$y_1=b+\lambda I^{\alpha}y_0(x)+I^{\alpha}f(x)=\mathrm{b}+\frac{\lambda}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}b\,dt+I^{\alpha}f(x)=$$

$$=b+\frac{\lambda b}{\Gamma(\alpha)}\frac{1}{\alpha}(\mathrm{x}-\mathrm{t})^{\alpha}\Big|_0^x+I^{\alpha}f(x)=b+\frac{\lambda b x^{\alpha}}{\alpha\Gamma(\alpha)}+I^{\alpha}f(x)=$$

$$=b+\frac{\lambda b x^{\alpha}}{\Gamma(\alpha+1)}+I^{\alpha}f(x);$$

To the above similar $y_2(x)$ also for the following the formula writing we get :

$$y_2=b+\lambda I^{\alpha}y_1(x)+I^{\alpha}f(x)=$$

$$=\mathrm{b}+\frac{\lambda}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}\left[b+\frac{\lambda b t^{\alpha}}{\Gamma(\alpha+1)}+I^{\alpha}f(\mathrm{t})\right]dt+I^{\alpha}f(x)=$$

$$=b+\frac{\lambda b}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}dt+\frac{\lambda^2 b}{\Gamma(\alpha+1)\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}t^{\alpha}dt+$$

$$+\frac{\lambda}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1} I^{\alpha} f(\mathrm{t})dt + I^{\alpha} f(x) =$$

$$= b - \frac{\lambda b}{\alpha \Gamma(\alpha)} (\mathrm{x}-\mathrm{t})^{\alpha}\Big|_0^x + \frac{\lambda^2 b x^{2\alpha}}{\Gamma(\alpha+1)\Gamma(\alpha)} B(\alpha,\alpha+1) +$$

$$+\lambda I^{\alpha}\left[I^{\alpha} f(x)\right] + I^{\alpha} f(x) = b + \frac{\lambda b x^{\alpha}}{\Gamma(\alpha+1)} + \frac{\lambda^2 b x^{2\alpha}}{\Gamma(2\alpha+1)} +$$

$$+\lambda I^{\alpha}\left[I^{\alpha} f(x)\right] + I^{\alpha} f(x) = b \cdot \sum_{k=0}^{2} \frac{\lambda^k x^{2\alpha}}{\Gamma(\alpha k+1)} + \lambda I^{2\alpha} f(x) + I^{\alpha} f(x);$$

The process continue bringing the following sequence harvest we do :

$$y_m(x) = b \cdot \sum_{k=0}^{m} \frac{(\lambda x^{\alpha})^k}{\Gamma(\alpha k+1)} + \sum_{k=1}^{m} \lambda^{k-1} I^{\alpha k} f(x) =$$

$$= b \cdot \sum_{k=0}^{m} \frac{(\lambda x^{\alpha})^k}{\Gamma(\alpha k+1)} + \int_0^x \sum_{k=1}^{m} \frac{\lambda^{k-1}}{\Gamma(\alpha k)} (x-t)^{\alpha k-1} f(t)dt;$$

m in this sequence to infinity yearning to the limit if we pass (2.11) of the integral equation to the solution we will come

$$y(x) = b \cdot \sum_{k=0}^{\infty} \frac{(\lambda x^{\alpha})^k}{\Gamma(2\alpha+1)} + \int_0^x (x-t)^{\alpha-1} \sum_{k=0}^{\infty} \frac{(\lambda (x-t)^{\alpha})^i}{\Gamma(\alpha i+\alpha)} f(t)dt. \quad (2.12)$$

Now this is the solution of (2.12). appearance condense for citation in chapter 1 passed

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+\beta)} \quad (2.11)$$

Mittag-Leffler from the function we use Necessary in places this function expression if we replace it , the following in appearance to the solution have we will be :

$$y(x) = bE_{\alpha,1}\left[\lambda x^{\alpha}\right] + \int_0^x (x-t)^{\alpha-1} E_{\alpha,\alpha}\left[\lambda (x-t)^{\alpha}\right] f(t)dt. \quad (2.13)$$

Example 2. Caputo's meaning to us Koshi type problem is given be :

$$({}^{c}D^{\frac{1}{2}}\,\mathrm{y})(x) + 2y(x) = 10, \quad y(+0) = b \;\; b \in \sim \quad (2.14)$$

$0 < \alpha < 1 \;\; va \;\; \lambda \in \sim$ when find the solution .

Equation (2.14). for those given writing we can

$\alpha = \frac{1}{2}, \quad f(x) = 10$, b=3 , λ=-2

Above proven (2.13) to the formula reached let's leave

$$y(x)=3E_{\alpha,1}(-2x^{\alpha})+\int_0^x (x-t)^{\alpha-1}E_{\alpha,\alpha}(-2(x-t)^{\alpha})f(t)dt=$$

$$=3E_{\frac{1}{2}}(-2x^{\frac{1}{2}})+10\int_0^x (x-t)^{-\frac{1}{2}}E_{\frac{1}{2},\frac{1}{2}}(-2(x-t)^{\frac{1}{2}})dt=$$

$$=3E_{\frac{1}{2}}(-2x^{\frac{1}{2}})+10x^{\frac{1}{2}}E_{\frac{1}{2},\frac{3}{2}}(-2x^{\frac{1}{2}}).$$

3- example _ Caputo's meaning to us Koshi type problem is given be :

$$({}^{c}D_{a+}^{\alpha}\,\mathrm{y})(x)-\lambda y(x)=f(x),\quad y(a+)=b\ \ (b\in\sim) \tag{2.11}$$

$0<\alpha<1\ \ va\ \ \lambda\in\sim$ when find the solution .

$$y(x)=bE_{\alpha}\left[\lambda(x-a)^{\alpha}\right]+\int_a^x (x-t)^{\alpha-1}E_{\alpha,\alpha}\left[\lambda(x-t)^{\alpha}\right]f(t)dt. \tag{2.12}$$

solution in appearance will be

Example 4 . Eq one sexual has been in case

$$({}^{c}D_{a+}^{\alpha}\,\mathrm{y})(x)-\lambda y(x)=0,\quad y(a)=b\ \ (b\in\sim) \tag{2.13}$$

solution as follows will be :

$$y(x)=bE_{\alpha}\left[\lambda(x-a)^{\alpha}\right] \tag{2.14}$$

Example 5. $({}^{c}D_{a+}^{\frac{1}{2}}\,\mathrm{y})(x)-\lambda y(x)=f(x),\quad y(a+)=b\ \ (b\in\sim)$

this equation the solution the following in appearance will be

$$y(x)=bE_{\frac{1}{2}}\left[\lambda(x-a)^{\frac{1}{2}}\right]+\int_a^x (x-t)^{-\frac{1}{2}}E_{\frac{1}{2},\frac{1}{2}}\left[\lambda(x-t)^{\frac{1}{2}}\right]f(t)dt.$$

One sexual has been case for looking after if we

$$({}^{c}D_{a+}^{\frac{1}{2}}\,\mathrm{y})(x)-\lambda y(x)=0,\qquad y(a)=b\ \ (b\in\sim)$$

solution $y(x)=bE_{\frac{1}{2}}\left[\lambda(x-a)^{\frac{1}{2}}\right]$ in appearance will be

Example 6. This example order high has been case for seeing we go out

$$({}^{c}D_{a+}^{\alpha}\,\mathrm{y})(x)-\lambda y(x)=f(x),\quad y(a)=b,\ \ y'(a)=d \tag{2.15}$$

Koshi of the issue $1<\alpha<2\ \ va\ \ \lambda, b, d\in\sim$ when the solution

$$y(x)=bE_{\alpha}\left[\lambda(x-a)^{\alpha}\right]+d(x-a)\ E_{\alpha,2}\left[\lambda(x-a)^{\alpha}\right]+$$

$$+\int_a^x (x-t)^{\alpha-1}E_{\alpha,\alpha}\left[\lambda(x-t)^{\alpha}\right]f(t)dt \tag{2.16}$$

with the formula is expressed .

If Eq one sexual

$$({}^{c}D_{a+}^{\alpha}\,y)(x)-\lambda y(x)=0,$$
$$y(a)=b,\quad y'(a)=d \tag{2.17}$$

Koshi of the issue $1<\alpha<2$ *va* $\lambda, b, d \in$ ~ when the solution

$$y(x)=bE_{\alpha}\left[\lambda(x-a)^{\alpha}\right]+d(x-a)\,\mathrm{E}_{\alpha,2}\left[\lambda(x-a)^{\alpha}\right] \tag{2.18}$$

with the formula is expressed .

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