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Identifying Key-Factors Driving Platform Ecosystem to Collapse

Lamia Loudahi

College of Mathematics and Computer Science, Zhejiang Normal University, Zhejiang, China

ABSTRACT

Many researchers in many fields have experienced tipping point to their complex systems (as in financial markets, in ecological system), but no one has experienced in platforms ecosystem systems. One of the biggest issues those platforms ecosystem can face: is the collapse. The risk of approaching to the tipping point is unknown. Complex dynamical system ranging from ecosystem to economy can collapse anytime, but predicting the point where the collapse can occur is difficult. We built a mathematical model (S-C model) which incorporating the dynamics, interactions and mutualistic network for platform ecosystem. We use this model to predict the key factors driving platforms ecosystem to collapse. To get our predictions we used an approximation method to get rid from complexity without losing much generality and still explain the same dynamics. To achieve our results we used matlab software and solved the reduced model. The inevitable factors that lead to collapse are suggesting to be used as early indicators of dramatic changes. We developed a system dynamics model of platform (group of suppliers and consumers) that includes: growth, churn, competition, alliance, negative interaction and mutual interaction between users. We use this model to simulate various development paths by varying different factors, which affect the platform's ecosystem model. Our simulation results show that: d_s, d_c, B_{ij}, λ_{ij}, and h are the key factors driving the platform ecosystem to collapse.

KEYWORDS: *Platform ecosystem, collapse, Interaction, Network, Consumers*

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I. INTRODUCTION

There is now an emerging need to show performance in real-time, as more and more customers are looking for nimble and secure platforms that can be agile enough to move grow and change with them. Platforms are among the most successful business models. However, not all platforms are successful; rather, many platforms fail because they do not attract enough users and do not solve the main challenge of reaching the tipping point [1].In recent years many efforts have been made to understand the mechanism of sudden collapse due to small changes that happened in the system. Tipping points depend on the existence of multiple distinct stable fixed points [14]. Complex and nonlinear ecological networks can exhibit a tipping point at which a transition to a global extinction state occurs [2]. The point at which a system can move from one stable state into another is called tipping point (or threshold) [8]. When the instability comes after stability it creates a tipping point in any system (disequilibrium) [9] [12]. This critical point may come because of small changes in the system [9]. A tipping point is where a small intervention leads to large and long-term consequences for the evolution of a complex system [13]. Complex dynamical systems, ranging from ecosystems to financial markets and the climate, can have tipping points at which a sudden shift to a contrasting dynamical regime may occur [7]. Many important ecosystems may currently be threatened with collapse [3] like platform ecosystem. Nonlinear stochastic complex networks in ecological systems can exhibit tipping points [5]. The moment at which sudden change occurs in complex networked systems may offer insights that prevent colony collapse disorder [7]. Complex networked systems ranging from ecosystems and the climate to economic, social, and infrastructure systems can exhibit a tipping point at which a total collapse of the system occurs [4]. To predict tipping point is an outstanding and extremely challenging problem. Tipping points are difficult to anticipate or detect. The initial change may be gradual but then rapidly accelerate as a tipping point is approached. Nonlinearly responding systems can change disproportionally to small changes in stressors with little or no warning [6]. It is becoming increasingly clear that many complex systems have critical thresholds so-called tipping points at which the system shifts abruptly from one state to another[7]. Critical slowing down appears to be generic for a wide class of local bifurcations (7), at which the current equilibrium state of a system loses stability before being replaced by another equilibrium state [9]. The theoretical study of two-sided markets began gaining attention in the early 2000s [18,19,20,21,22,23,24]

This paper defines a new math model of the platform ecosystem; we take help from a Lotka-Voltera model based on [15, 16, 17]. We build up a large number of connected users (set of interacting elements). Our model represents both suppliers and consumers growth, suppliers and consumers churn, competition within consumers (interspecific and intraspecific), the alliance within suppliers, the negative interaction within consumers and the mutual interaction between suppliers and consumers this model developed based on the Lotka-Voltera one. The competition within the platform can be compared to predators and prey; the difference is that the competing companies are predators and preys simultaneously. Each one tries to be the best and the market leader. In [25] Guofu Tan and Junjie Zhou said that if a platform competition increase, prices and platform profits can increase while consumers surplus can decrease. While the alliance can be compared to plants and pollinators, the plant needs pollinators either pollinators do not need the plant. However, in the alliance of companies representing mutual benefit, each one needs the others. The negative interaction within consumers bans the platform from growing, for example if a consumer "I" share his bad experience in the platform with other consumers, he helps the others to do not fall in the same bad situation, but this act will cause a very dangerous damages for the platform, many consumers will drop out from the platform and find a suitable one.

II. METHODOLOGY

The model used in this research is a pair of non-linear differential equations of the first order. It describes the dynamics of our system, including different interactions. The model developed in this study is based on the Lotka-Voltera approach, which also a couple of differential equations of first order but used to describe the dynamics of a biologic system (predator-prey). The evolution of the platform ecosystem used in this study is as follow:

$$\begin{cases} \dfrac{dS_i}{dt} = S_i\left(r_i - \sum_{j\in M_1} B_{ij}S_j + \sum_{j\in M_1} \delta_{ij}S_j + \dfrac{\sum_{k=1}^{M_2}\gamma_{ik}^{(S_i)}C_k}{1+h\sum_{k=1}^{M_2}\gamma_{ik}^{(S_i)}C_k} \right) \\ \dfrac{dC_i}{dt} = C_i\left(\mu_i - \sum_{j\in M_1} \lambda_{ij}S_j + \dfrac{\sum_{k=1}^{M_1}\gamma_{ik}^{(C_i)}S_k}{1+h\sum_{k=1}^{M_1}\gamma_{ik}^{(C_i)}S_k} \right) \end{cases} \quad (1)$$

Where S_i and C_i are the numbers of suppliers and consumers, respectively, r_i represent the growth rate for each supplier in group i (without interspecific and interspecific competition). This rate (r_i) is the percentage change in the number of vendors during the time. (μ_i) is the growth rates of consumers. (B_{ij}) represents the coefficient of competition between the supplier in group i and group j. Whereas (μ_{ij}) is the coefficient of mutualistic interaction (alliance or partnership) between suppliers in group i and in group j. (λ_{ij}) is the coefficient of the negative interaction within consumers $\gamma_{ik}^{(S_i)}$ and $\gamma_{ik}^{(C_i)}$ while, h is the half-saturation point (is a constant which limits the number of suppliers and consumers).

$$\begin{cases} \dfrac{dS_{eff}}{dt} = (r_1 - d_s)S_{eff} - BS_{eff}^2 + \delta S_{eff}^2 + \dfrac{\langle \gamma^{(S_i)} \rangle C_{eff}}{1+h\langle \gamma^{(S_i)} \rangle C_{eff}} S_{eff} \\ \dfrac{dC_{eff}}{dt} = (\mu_1 - d_c)C_{eff} - \lambda C_{eff}^2 + \dfrac{\langle \gamma^{(C_i)} \rangle S_{eff}}{1+h\langle \gamma^{(C_i)} \rangle S_{eff}} C_{eff} \end{cases} \quad (2)$$

Where the dynamical variables S_{eff} and C_{eff} are effective or the average number of suppliers and consumers, respectively. (r_1) and (μ_1) are effective growth rates for suppliers and consumers, respectively. B is the parameter that characterized the effect of the intraspecific and interspecific competition; δ and λ are the parameters that characterize the effects of an intraspecific and interspecific alliance of suppliers and negative interaction within consumers (respectively). $\langle \gamma_{ij}^{(S_i)} \rangle$ And $\langle \gamma_{ij}^{(C_i)} \rangle$ are the effective mutualistic strength associated with the suppliers and consumers, respectively. We have $r = r_1 - d_s$, and $\mu = \mu_1 - d_c$. Where r_1 and μ_1 are the suppliers' and consumers' subscription rates (respectively), while d_s and d_c are the suppliers' and consumers' churning rates (respectively).

$$\begin{cases} \dfrac{dS_{eff}}{dt} = (r_1 - d_s)S_{eff} - BS_{eff}^2 + \delta S_{eff}^2 + \dfrac{\langle \gamma^{(S_i)} \rangle C_{eff}}{1+h\langle \gamma^{(S_i)} \rangle C_{eff}} S_{eff} = 0 \\ \dfrac{dC_{eff}}{dt} = (\mu_1 - d_c)C_{eff} - \lambda C_{eff}^2 + \dfrac{\langle \gamma^{(C_i)} \rangle S_{eff}}{1+h\langle \gamma^{(C_i)} \rangle S_{eff}} C_{eff} = 0 \end{cases} \quad (3)$$

From equation (3) we can see that (0, 0) is one of the roots of this equation, the other root can be written as follows:

$$\begin{cases} (r_1 - d_s) + (\delta - B) S_{eff} + \dfrac{\langle \gamma^{(S_i)} \rangle C_{eff}}{1 + h \langle \gamma^{(S_i)} \rangle C_{eff}} = 0 \\ (\mu_1 - d_c) - \lambda C_{eff} + \dfrac{\langle \gamma^{(C_i)} \rangle S_{eff}}{1 + h \langle \gamma^{(C_i)} \rangle S_{eff}} = 0 \end{cases} \quad (4)$$

We can write also:

$$\begin{cases} \left[(r_1 - d_S) + \dfrac{\langle \gamma^{S_i} \rangle C_{eff}}{1 + h \langle \gamma^{S_i} \rangle C_{eff}} \right] (B - \delta)^{-1} \\ \left[(\mu_1 - d_C) + \dfrac{\langle \gamma^{C_i} \rangle S_{eff}}{1 + h \langle \gamma^{C_i} \rangle S_{eff}} \right] \lambda^{-1} \end{cases} \quad (5)$$

We have $r = r_1 - d_s$ and $\mu = \mu_1 - d_c$. Where r_1 and μ_1 are the suppliers' and consumers' subscription rates, while d_s and d_c are the suppliers' and consumers' churning rates.

The solutions of Eq.4 can be conveniently expressed in terms of the following quadratic equation for (S_{eff}) and (C_{eff}):

$$\begin{cases} q_1 S_{eff}^2 + q_2 S_{eff} + q_3 = 0 \\ p_1 C_{eff}^2 + p_2 C_{eff} + p_3 = 0 \end{cases} \quad (6)$$

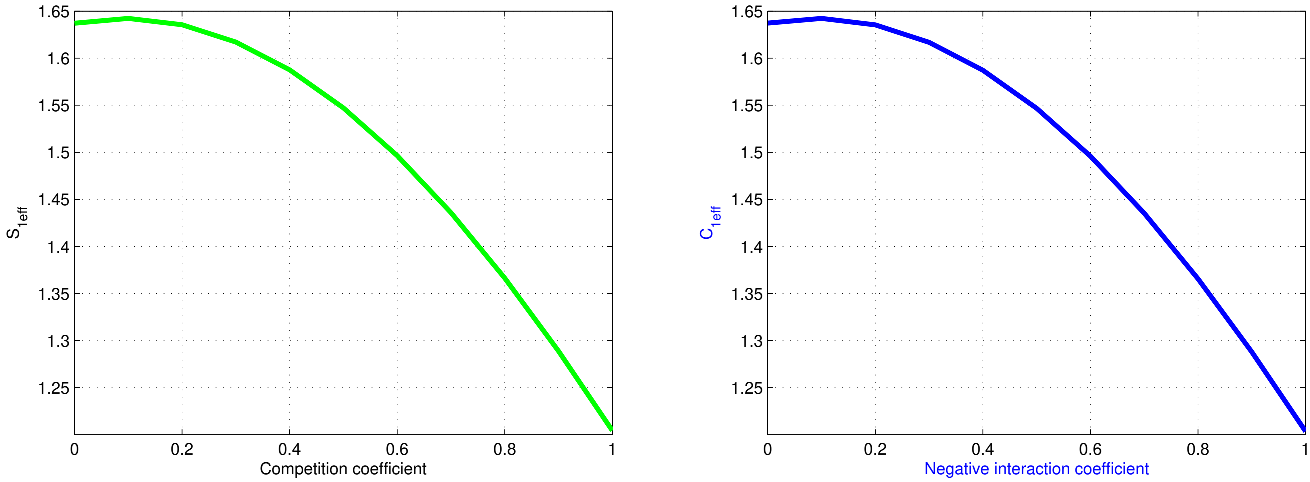

Figure 1: The effect of competition coefficient (within suppliers) and negativeinteraction (within consumers) on the tipping point. These graphs show the shut-ting down of equilibrium point. Competition and negative interaction are key factors that increasingly lead to collapse. For each network, the parameter values are: $\langle \gamma_S \rangle = \langle \gamma_C \rangle = 1$, $r_1 = 0.3$, $\mu_1 = 0.3$, $\delta = 0.1$, $h = 0.7$, $d_s = 0$ andd_c= 0

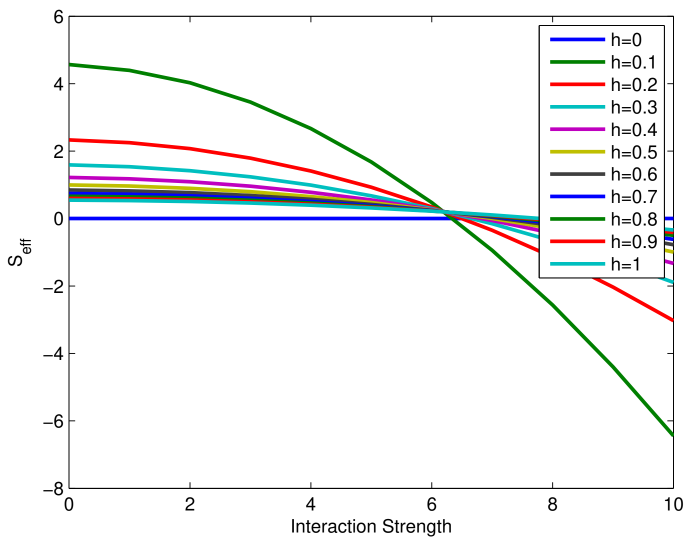

Figure 2: The effect of the interaction strength coefficient on the number ofconsumers and suppliers. For each network, the parameter values are: $\boldsymbol{B}_{ii} = 1$,

$\boldsymbol{B}_{ij} = 0.5$ $r_1 = 0.3$, $\delta = 0.1$, $\boldsymbol{d}_s = 0.1$

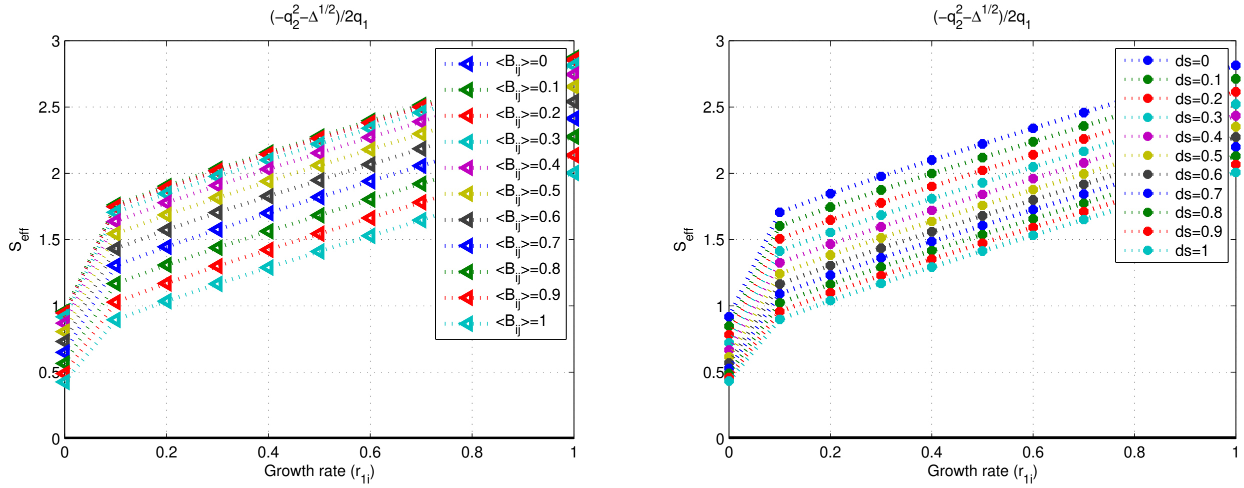

Figure 3: The effect of the growth rate on the tipping point (Supplier side). Foreach network, the parameter values are for left graph are: $\langle \gamma_S \rangle = \langle \gamma_C \rangle = 1$, $\delta = 0.1$, $h = 0.5$, $d_s = 0$ $\boldsymbol{B}_{ii} = 1$. For right graph are: $\langle\gamma_S\rangle = \langle \gamma_C \rangle = 1$, $\delta = 0.1$, $h = 0.5$, $\boldsymbol{B}_{ii} = 1$ and $\boldsymbol{B}_{ij} = 0.3$

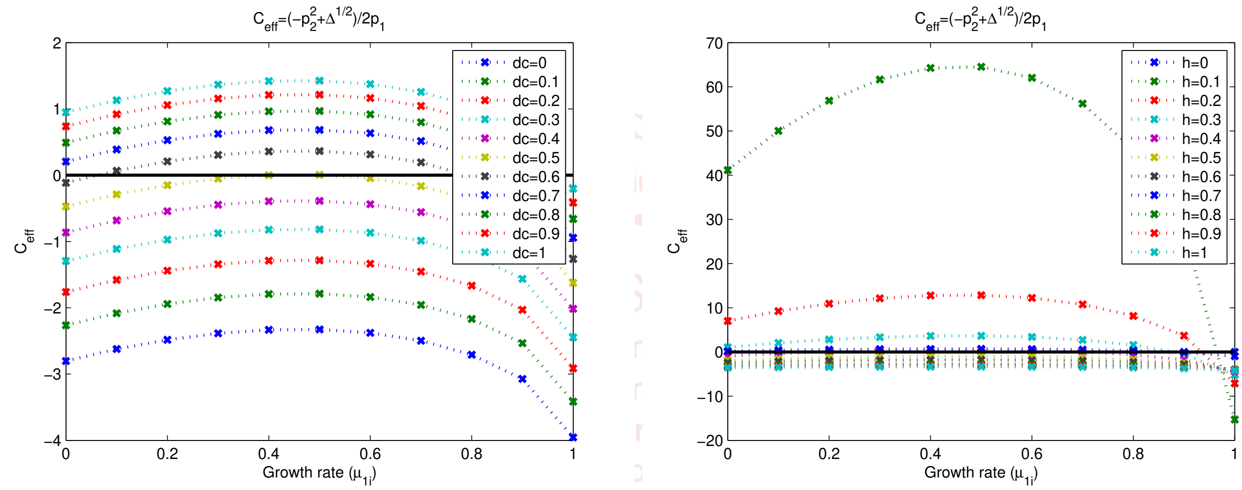

Figure 4: The effect of the growth rate on the tipping point (Consumer side). For each network, the parameter values of left graph are: $\langle \gamma_S \rangle = \langle \gamma_C \rangle = 1$, $h = 0.7$ and $\lambda_{ij} = 0.1$. For right graph are: $\langle \gamma_S \rangle = \langle \gamma_C \rangle = 1$, $d_c = 0$ and $\lambda_{ij} = 0.1$.

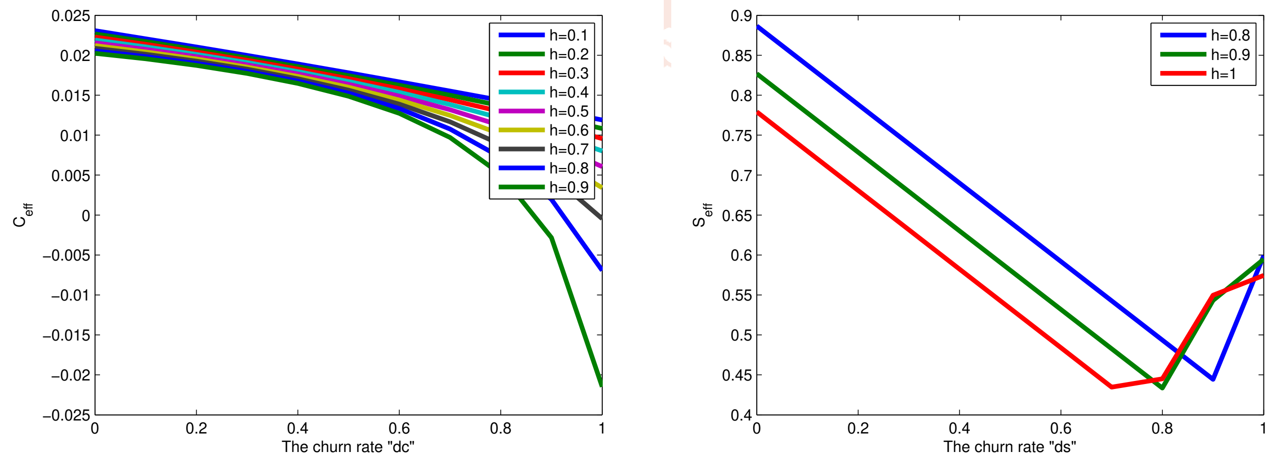

Figure 5: The effect of the churning rates on the number of consumers and suppliers. For each network, the parameter values are: $\mu_1 = 0.15$, $\lambda_{ii} = 1$, $\lambda_{ij} = 0$ and $l = 0.5$ but in the second graph for Seff we have $r_1 = 0.7$, l=0.5, B_{ii}=*1*, B_{ij}=0.

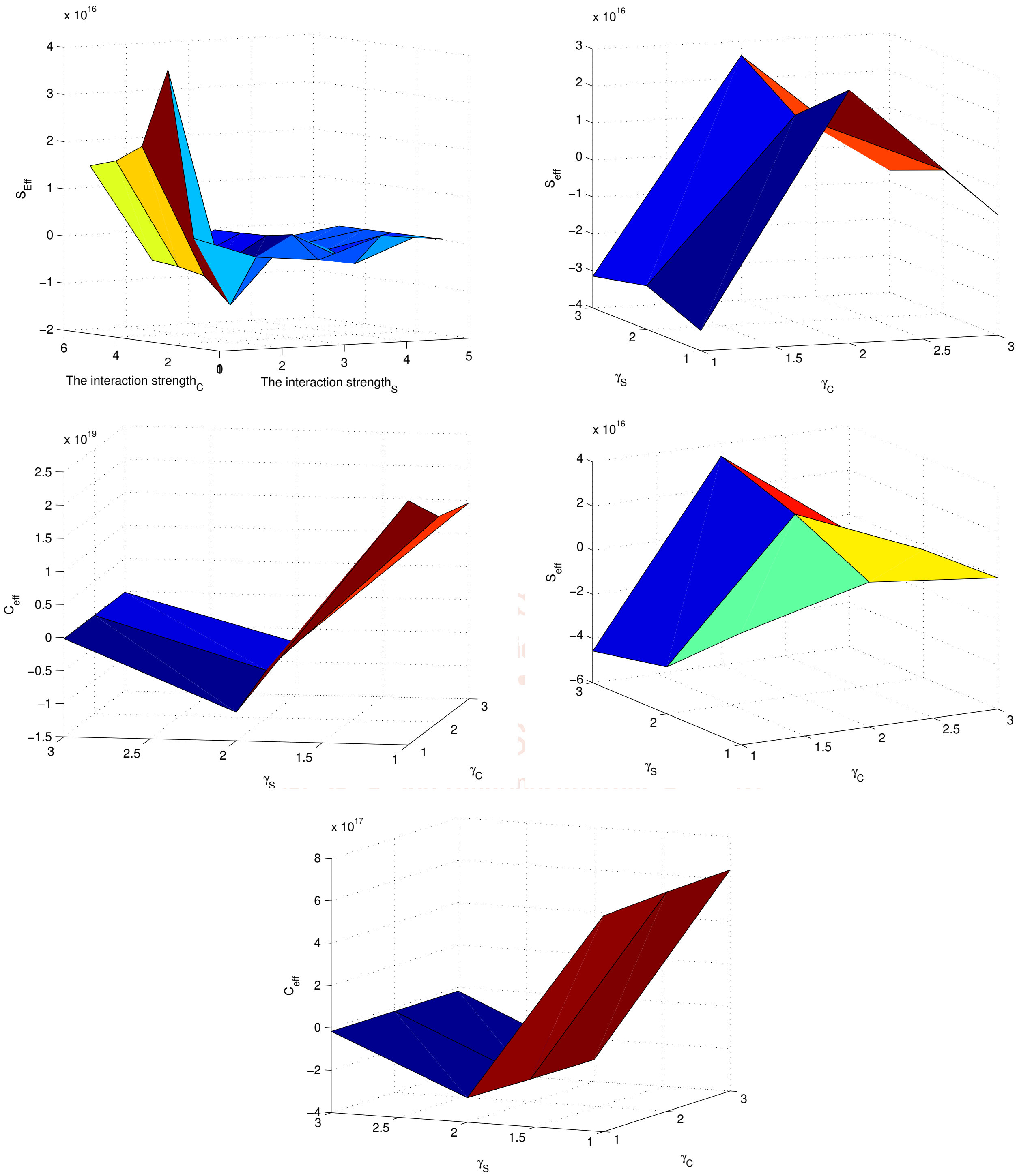

Figure 6: The effect of the interaction strength on the number of consumers and suppliers (3D graph). For each network we have: first graph from the left we fixed $\delta_{ij} = 0$, $B_{ij} = 0$, $r_1 = 0.3$, $d_s = 0.1$, and $h = 0.2$. Second graph to the right we have: $\delta_{ij} = 0.1$, $B_{ij} = 0.1$, $r_1 = -0.3$, $d_s = 0.1$, and $h = 0.2$. Third graph we have: $\lambda_{ij} = 0.1$, $\mu_1 = -0.3$, $d_c = 0.1$, and $h = 0.2$. Fourth graph we have: $\delta_{ij} = 0.1$, $B_{ij} = 0.1$, $r_1 = 0.3$, $d_s =0.1$, and $h = 0.2$. The last graph we have: $\lambda_{ij} = 0.1$, $\mu_1 = 0.3$, $d_c = 0.1$, and $h = 0.2$.

III. DISCUSSION AND RESULTS

In this work we present an approach to measure the points of collapse for suppliers and consumers. The proposed model incorporates growth, internal perturbation (churning), competition, alliance, and mutualistic interaction between suppliers and consumers. As a result of management, we suppose that there are economic changes, so that some parameters will increase while some others will decrease, rendering inevitable eventual collapse of our system. From the figures we made, we can conclude the factors that lead to collapse. We have adopted an approximation theory to understand the mechanism of the system and controlling its equilibrium point. In particular we aim to predict the factors that cause the collapse. The basic idea is to find the point from which our system switches from a high stable steady state to a low stable steady state. We present results with 6 figures, we vary γ_S, γ_C , d_s, d_c, r_1, μ_1, B_{ij}, δ_{ij}, λ_{ij}, h and l, systematically and study whether a reduction of these parameters may lead to a tipping point.

In figure1 the parameters are fixed: $h = 0.7$; $\langle\gamma^S\rangle=\langle\gamma^C\rangle= 1$; $r_1 = \mu_1 = 0.3$; $\delta = 0.001$; $B_{ij} \epsilon [0, 1]$, $\lambda \epsilon [0, 1]$ and $d_s=0$, these 2 graphs show the shutting down of the equilibrium point. The effect of competition is increasingly leading to fast collapse, and especially for S_{eff} and C_{eff} we can see clearly the change down. Green line shows how the number of suppliers goes down, due to the rising of the competition coefficient B_{ij}, while the blue line shows the decreasing of the number of consumers due to the rising of λ_{ij}. The result shows that our model containing at least two key factors (B and λ) that can provoke the fast decreasing number of users in the platform whenever the interspecific competition B_{ij} and the negative interaction (interspecific) λ_{ij} are increasing, the equilibrium tends to collapse. (It's unwanted that B_{ij} and λ_{ij} increase, also some others parameters value are affecting positively the collapse; but we can't discuss them in only one figure, for that we made other figures that make things more clear. In figure 2 the parameters $\langle\gamma^S\rangle \in [0, 10]$, $\langle\gamma^C\rangle \epsilon [0, 10]$; $B_{ii} =1$, $B_{ij} =0.5$, $\delta=0.1$ and $d_s=0.1$. We presented in many lines with different colors the movements of S_{eff} in terms of interaction strength, we can see when $h=0$ (the blue line) and when $h=0.1$ (green line), the S_{eff} decreases faster compare to the other lines, whenever h is rising the S_{eff} slowly decrease. We increased the value $B_{ij} = 0.5$ and we can see that mall change in d_s and d_c will greatly change our results, these values and h are affecting negatively the steady state, Seff and C_{eff} decrease to a negative values, we see the point where total collapse occur *(a, b)* where $a \epsilon [6, 8]$ and $b \epsilon [0, 1]$. Here we took the churning rate non zero, which means that the growth rate will automatically decrease, not only d_c and d_s but also the interspecific competition, here it is equal to 0.5, this value is big while δ_{ij} is too small which makes the collapse easier. In figure 3 we fixed for the left graph $B_{11} = 1$, $h = 0,5$; δ= 0:1, $\langle\gamma^S\rangle=\langle\gamma^C\rangle= 1$, $d_s=0$ and $r_1 \epsilon [-1;-0,3]$. For the right graph we have fixed $B_{ii} = 1$, $h = 0.5$, δ= 0:1, $\langle\gamma^S\rangle=\langle\gamma^C\rangle= 1$, $B_{ij} = 0.3$ and $r_1 \epsilon [-0.2; 1]$. In the interval [-1;-0,3] we have $\Delta < 0$, but in the interval [-0:2; 1] we have $\Delta > 0$ In the two graphs we see that the number of suppliers is increasing whenever we're rising the growth rate, from the figure we notice that $B_{ij} < B_{i+1,j}$ or $d_{s,i}<d_{s,i+1}$ we have $(S_{eff})_i < (S_{eff})_{i+1}$ *(i=0,0.1,0.2,…,1)* which affect negatively the growth of the number of suppliers. In figure 4 for the left graph we fixed $\lambda = 0.1$; $\langle\gamma^S\rangle=\langle\gamma^C\rangle= 1$; $h=0.7$ and $d_c=\{0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9\}$, while for right graph we fixed: $\langle\gamma^S\rangle=\langle\gamma^C\rangle= 1$; $d_c=0$ and $h=\{0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9\}$ In this figure we see the decreasing of C_{eff} even we are rising the growth rate cue to other internal or external factors and aslo the nature of discriminant. d_c drive the system to collapse faster, while h is driving to the fast collapse if it is approaching to zero. Figure 3 shows the 4 solutions which are depending on: Δ_s, Δ_c, r_1, μ_1, B_{ij}, h and d_s, d_c; here we have two increasing solutions and two decreasing ones, we note that B_{ij} interact as the same way as ds. In figure 4 we concentrated on the consumers results, we have 2 sgraphs depending on: Δ_c, d_c, μ_1 and h. We can see the difference on the results when changing the values of d_c, and h. In figure 5 we took on the x-abscissa d_s and d_c. we fixed $\mu_1 = 0.15$, $\lambda_{ii} = 1$, $\lambda_{ij} = 0$, $l = 0.5$, $r_1 = 0.7$, $l=0.5$, $B_{ii}=1$, $B_{ij}=0$. and the solutions are decreasing faster when d_c are increasing and h decreasing, these factors causing a faster collapse. In the 2nd graph, the solutions are decreasing then increasing these change is due to the big value of growth rate, here for h we have only three values, if h is less than 0.8 the discriminant is negative which lead to a complex solution which is physically unrealistic; if we read this graph conversely we see tipping points at $d_s \epsilon [0.6, 1]$. We summary in this figure that when the value of d_s and d_c are increasing from 0 to 1, S_{eff} moves continuously (decreasing) in the negative side (for first graph in the left side), and moves continuously (decreasing then increasing) in the positive side (second graph in the left side), also C_{eff} moves continuously form positive side to negative side (the graph in the right side. We passed to a 3D plot in figure 6, it shows the effect of the interaction strength on the tipping point, and we can see the tipping point in each graph which is depending on the values of system's parameters and the nature of the discriminant Δ_s and Δ_c. Some graphs are increasing some others are decreasing due to the nature of Δs and Δc.

From results found in [4] we can conclude that if $r_1 > 0$; $\mu_1 > 0$, and $d_s = 0$; $d_c= 0$ we have: $q_1 < 0$, $p_1 < 0$, $q_3 < 0$, and $p_3 < 0$ in this case we have two solutions (positive and negative solutions). In the case $r_1 < 0$; $\mu_1 < 0$ and for $d_s = 0$; $d_c= 0$ we have:

$$\left[(r_1-d_S)+\frac{\langle\gamma^{S_i}\rangle C_{eff}}{1+h\langle\gamma^{S_i}\rangle C_{eff}}\right]>0 \text{ and } \left[(\mu_1-d_C)+\frac{\langle\gamma^{C_i}\rangle S_{eff}}{1+h\langle\gamma^{C_i}\rangle S_{eff}}\right]>0$$

In this case the solutions are:

$$\begin{cases} S_{1eff}=0 \\ S_{2eff}=\left[(r_1-d_S)+\dfrac{\langle\gamma^{S_i}\rangle C_{eff}}{1+h\langle\gamma^{S_i}\rangle C_{eff}}\right](B-\delta)^{-1} \end{cases} \qquad (7)$$

$$\begin{cases} C_{1eff} = 0 \\ C_{2eff} = \left[(\mu_1 - d_C) + \dfrac{\langle \gamma^{C_i} \rangle S_{eff}}{1 + h\langle \gamma^{C_i} \rangle S_{eff}} \right] (\lambda)^{-1} \end{cases} \quad (8)$$

In the case $S_{eff} = 0$ and $C_{eff} = 0$, it corresponds to the churning state.

In the case $q_2^2 - 4q_1q_3 < 0$, and $p_2^2 - 4p_1p_3 < 0$ the solution is complex, and physically it is unrealistic we consider it unrealistic in our research.

In all these figures we use our framework to identify the changes in the parameter's values and intervals that are associated with a collapse from a state of users subscriptions (i.e. $S_{eff} > 0$, $C_{eff} > 0$), to a state where all users will unsubscribe (i.e. $S_{eff} \leq 0, C_{eff} \leq 0$).

IV. CONCLUSION

Platforms have become one of the most important business models of the 21st century, but many of them fail. Platforms are built on a variety of factors, but often fail because of the mismanagement of those factors .To understand why and how platforms fail, we tried to identify the key factors deriving platforms to collapse. From the figures we've made we general lessons about why platforms struggle. After analyzing our model we notice that platforms fail for these reasons: maximizing the churning rates (d_s, d_c), maximizing the intraspecific competition (B_{ij}), maximizing the negative interactions (λ_{ij}), and minimizing the half saturation (h), Since many things can go wrong in a platform. If a firm cannot stay competitive, no market position is safe. By identifying the sources of failure, managers can reduce the mistakes that lead to failure.

V. SUPPLEMENTARY MATERIAL FOR: "Identifying key factors driving platform ecosystem to collapse".

1. INTRODUCTION

There are several types of platforms such as Amazon, EBay, Google, Alibaba, Aliexpress, Taobao...etc; The notion of platform has been developed by management researchers in three different domain of research (product, technological system, and transactions)[1]. Platforms are often associated with the "network Effects": that is, the more users adopt the platform, the more valuable the platform becomes to the owner and users due to increasing access to the network of users and often to a growing set of complementary innovations In other words, there are more and more incentives to more companies and users to adopt the platform and join Ecosystem with the arrival of more users and add-ons [2].

2. DERIVATION OF THE 2D REDUCED MODEL

First of all from eq.4, we can obtain the effective average number of suppliers and consumers and we can write:

$$r_i S_i \approx r\, S_{eff} \quad (9)$$

$$r_i = r_{i,1} - d_{i,s} \quad (10)$$

$$(r_{i,1} - d_{i,s}) S_i = (r_1 - d_s) S_{eff} \quad (11)$$

And

$$\mu_i C_i \approx \mu\, C_{eff} \quad (12)$$

$$\mu_i = (\mu_{i,1} - d_{i,c}) \quad (13)$$

$$(\mu_{i,1} - d_{i,c}) C_i \approx (\mu_1 - d_c) C_{eff} \quad (14)$$

Here we define S_{eff} and C_{eff} as the effective number of suppliers and consumers (respectively).

Suppliers in different platforms and different commodities do not compete as those in same platform and same commodities, for that we can write: $B_{ii} >> B_{ij}$; also we can generate it for those who interact positively within them if they are in same commodities or different, so the positive interaction in same commodities will be stronger than that on in different commodities, and same situation for consumers, then we can write: $\delta_{ii} >> \delta_{ij}$ and $\lambda_{ii} >> \lambda_{ij}$. In other side we can write:

$$\sum_{j=1}^{M_1} B_{ij} S_j S_i \approx B\, S_{eff}^2 \qquad (15)$$

And

$$\sum_{j=1}^{M_1} \delta_{ij} S_j S_i \approx \delta\, S_{eff}^2 \qquad (16)$$

And

$$\sum_{j=1}^{M_2} \lambda_{ij} C_j C_i \approx \lambda\, C_{eff}^2 \qquad (17)$$

To integrate interspecific interaction in our model, we write the interactions terms as follow:

$$\sum_{j=1}^{M_1} B_{ij} S_j S_i \approx \frac{\sum_{i=1}^{M_1} \sum_{i=1}^{M_1} B_{ij}}{\sum_{i=1}^{M_1} 1} S_{eff}^2 \approx B\, S_{eff}^2 \qquad (18)$$

And

$$\sum_{j=1}^{M_1} \delta_{ij} S_j S_i \approx \frac{\sum_{i=1}^{M_1} \sum_{i=1}^{M_1} \delta_{ij}}{\sum_{i=1}^{M_1} 1} S_{eff}^2 \approx \delta\, S_{eff}^2 \qquad (19)$$

And

$$\sum_{j=1}^{M_2} \lambda_{ij} C_j C_i \approx \frac{\sum_{i=1}^{M_2} \sum_{i=1}^{M_2} \lambda_{ij}}{\sum_{i=1}^{M_2} 1} C_{eff}^2 \approx \lambda\, C_{eff}^2 \qquad (20)$$

Now for finding the effective interaction in the network of our model in both sides (suppliers side and consumers side), we start by calculating the strength of the mutualistic interaction for each group of suppliers and consumers as follows:

$$\sum_{j=1}^{M_2} \gamma_{ij}^{S_i} C_j \approx \sum_{j=1}^{M_2} \frac{\gamma_0}{(G_i)^l} \varepsilon_{ij} C_j \approx \gamma_0 G_i^{1-l} C_{eff} \qquad (21)$$

And

$$\sum_{j=1}^{M_1} \gamma_{ij}^{C_i} S_j \approx \sum_{j=1}^{M_1} \frac{\gamma_0}{(Z_i)^l} \varepsilon_{ij} S_j \approx \gamma_0 Z_i^{1-l} S_{eff} \qquad (22)$$

There are many ways and methods to get the average of the mutualistic strength, in this work we use the unweighted method and we find:

$$\left\langle \gamma_{ij}^{S_i} \right\rangle = \frac{\sum_{i=1}^{M_2} \gamma_0 G_i^{1-l}}{\sum_{i=1}^{M_2} 1} \qquad (23)$$

And

$$\left\langle \gamma_{ij}^{C_i} \right\rangle = \frac{\sum_{i=1}^{M_1} \gamma_0 Z_i^{1-l}}{\sum_{i=1}^{M_1} 1} \qquad (24)$$

3. STEADY STATE SOLUTION

To obtain the equilibrium point (steady state) solution of suppliers-consumers number from our reduced model is by solving two equations which are: $\frac{dS_{eff}}{dt} = 0$, and $\frac{dC_{eff}}{dt} = 0$ we have:

$$\frac{dS_{eff}}{dt}=(r_1-d_s)S_{eff}-BS_{eff}^2+\delta S_{eff}^2+\frac{\langle\gamma_{ij}^{(S_i)}\rangle C_{eff}}{1+h\langle\gamma_{ij}^{(S_i)}\rangle C_{eff}}S_{eff}=0 \qquad (25)$$

$$\frac{dC_{eff}}{dt}=(\mu_1-d_c)C_{eff}-\lambda C_{eff}^2+\frac{\langle\gamma_{ij}^{(C_i)}\rangle S_{eff}}{1+h\langle\gamma_{ij}^{(C_i)}\rangle S_{eff}}C_{eff}=0 \qquad (26)$$

As we can define the Jacobian matrix related to the equilibrium point solution in the way:

$$J=\begin{bmatrix}\frac{df}{dS_{eff}} & \frac{df}{dC_{eff}}\\ \frac{dg}{dS_{eff}} & \frac{dg}{dC_{eff}}\end{bmatrix} \qquad (27)$$

$$J=\begin{bmatrix} 2S_{eff}(\delta-B)+r_1-d_s+\frac{\langle\gamma_{ij}^{(S_i)}\rangle C_{eff}}{1+h\langle\gamma_{ij}^{(S_i)}\rangle C_{eff}} & \frac{\langle\gamma_{ij}^{(S_i)}\rangle\left(\left[S_{eff}+h\langle\gamma_{ij}^{(S_i)}\rangle S_{eff}C_{eff}\right]-C_{eff}h\langle\gamma_{ij}^{(S_i)}\rangle\right)}{\left(1+h\langle\gamma_{ij}^{(S_i)}\rangle C_{eff}\right)^2}\\ \frac{\langle\gamma_{ij}^{(C_i)}\rangle\left(\left[C_{eff}+h\langle\gamma_{ij}^{(S_i)}\rangle C_{eff}S_{eff}\right]-S_{eff}h\langle\gamma_{ij}^{(C_i)}\rangle\right)}{\left(1+h\langle\gamma_{ij}^{(S_i)}\rangle C_{eff}\right)^2} & -2\lambda C_{eff}+\mu_1-d_c+\frac{\langle\gamma_{ij}^{(C_i)}\rangle S_{eff}}{1+h\langle\gamma_{ij}^{(C_i)}\rangle S_{eff}}\end{bmatrix} \qquad (28)$$

(0, 0) is one of the roots of this equation, the other root can be written as follows:

$$\begin{cases}(r_1-d_s)+(\delta-B)S_{eff}+\frac{\langle\gamma^{(S_i)}\rangle C_{eff}}{1+h\langle\gamma^{(S_i)}\rangle C_{eff}}=0\\ (\mu_1-d_c)-\lambda C_{eff}+\frac{\langle\gamma^{(C_i)}\rangle S_{eff}}{1+h\langle\gamma^{(C_i)}\rangle S_{eff}}=0\end{cases} \qquad (29)$$

We can write also:

$$\begin{cases}\left[(r_1-d_S)+\frac{\langle\gamma^{S_i}\rangle C_{eff}}{1+h\langle\gamma^{S_i}\rangle C_{eff}}\right](B-\delta)^{-1}\\ \left[(\mu_1-d_C)+\frac{\langle\gamma^{C_i}\rangle S_{eff}}{1+h\langle\gamma^{C_i}\rangle S_{eff}}\right]\lambda^{-1}\end{cases} \qquad (30)$$

We have $r = r_1$-d_s and $\mu = \mu_1$-d_c. Where r_1 and μ_1 are the suppliers' and consumers' subscription rates, while d_s and d_c are the suppliers' and consumers' churning rates.

The solutions of Eq.4 can be conveniently expressed in terms of the followingquadratic equation for (S_{eff}) and (C_{eff}):

$$\begin{cases}q_1S_{eff}^2+q_2S_{eff}+q_3=0\\ p_1C_{eff}^2+p_2C_{eff}+p_3=0\end{cases} \qquad (31)$$

Where:

$$q_1=-\left(h\langle\gamma^{C_i}\rangle+h\langle\gamma^{S_i}\rangle\langle\gamma^{C_i}\rangle+h^2r_1\langle\gamma^{S_i}\rangle\langle\gamma^{C_i}\rangle\right) \qquad (32)$$

$$\begin{aligned}q_2=&(B-\delta)^2-h\ r_1(B-\delta)\langle\gamma^{S_i}\rangle+h\ r_1(B-\delta)\langle\gamma^{C_i}\rangle+\langle\gamma^{S_i}\rangle\langle\gamma^{C_i}\rangle+2\ h\ r_1\langle\gamma^{S_i}\rangle\langle\gamma^{C_i}\rangle\\ &+h^2r_1^2\langle\gamma^{S_i}\rangle\langle\gamma^{C_i}\rangle-d_s\left[h(B-\delta)\langle\gamma^{C_i}\rangle+h\ \langle\gamma^{S_i}\rangle\langle\gamma^{C_i}\rangle+h^2r_1\langle\gamma^{S_i}\rangle\langle\gamma^{C_i}\rangle\right]\end{aligned} \qquad (33)$$

$$q_3=r_1(B-\delta)+r_1\langle\gamma^{S_i}\rangle+h\ r_1^2\langle\gamma^{S_i}\rangle-d_s\left[(B-\delta)+h\ r_1\langle\gamma^{S_i}\rangle\right] \qquad (34)$$

And

$$p_1=-\left(h\langle\gamma^{S_i}\rangle+h\langle\gamma^{C_i}\rangle\langle\gamma^{S_i}\rangle+h^2\mu_1\langle\gamma^{C_i}\rangle\langle\gamma^{S_i}\rangle\right) \qquad (35)$$

$$\begin{aligned}p_2=&-(\lambda)^2-h\ \mu_1(\lambda)\langle\gamma^{S_i}\rangle+h\ \mu_1(\lambda)\langle\gamma^{S_i}\rangle+\langle\gamma^{C_i}\rangle\langle\gamma^{S_i}\rangle+2\ h\ \mu_1\langle\gamma^{C_i}\rangle\langle\gamma^{S_i}\rangle\\ &+h^2\mu_1^2\langle\gamma^{C_i}\rangle\langle\gamma^{S_i}\rangle-d_c\left[h(\lambda)\langle\gamma^{S_i}\rangle+h\ \langle\gamma^{C_i}\rangle\langle\gamma^{S_i}\rangle+h^2\mu_1\langle\gamma^{C_i}\rangle\langle\gamma^{S_i}\rangle\right]\end{aligned} \qquad (36)$$

$$q_3 = \mu_1(\lambda) + \mu_1\langle\gamma^{C_i}\rangle + h\,\mu_1^2\langle\gamma^{C_i}\rangle - d_c\left[(\lambda) + h\,\mu_1\langle\gamma^{C_i}\rangle\right] \qquad (37)$$

To find the other roots we have to solve equation 31; we have: To find the other roots we have to solve equation 6; we have: $\Delta_S = (q_2)^2 - 4q_1q_3$ and $\Delta_C = (p_2)^2 - 4p_1p_3$;

A. If $\Delta_S > 0$ and $\Delta_C > 0$ we have two solutions: $S_1 = \frac{-q_2 - \sqrt{\Delta_S}}{2q_1}$; $S_2 = \frac{-q_2 + \sqrt{\Delta_S}}{2q_1}$ and $C_1 = \frac{-p_2 - \sqrt{\Delta_C}}{2p_1}$; $C_2 = \frac{-p_2 + \sqrt{\Delta_C}}{2p_1}$.

B. If $\Delta_S = 0$ and $\Delta_C = 0$ we have a double solution: $S_1 = S_2 = \frac{-q_2}{2q_1}$ and $C_1 = C_2 = \frac{-p_2}{2p_1}$.

C. If $\Delta_S < 0$ and $\Delta_C < 0$: In this case the solution will be complex and physically unrealistic in our work.

D. If $\Delta_S > 0$ and $\Delta_C = 0$ we have: $S_1 = \frac{-q_2 - \sqrt{\Delta_S}}{2q_1}$; $S_2 = \frac{-q_2 + \sqrt{\Delta_S}}{2q_1}$ and $C_1 = C_2 = \frac{-p_2}{2p_1}$

E. If $\Delta_S > 0$ and $\Delta_C < 0$, we have: $S_1 = \frac{-q_2 - \sqrt{\Delta_S}}{2q_1}$; $S_2 = \frac{-q_2 + \sqrt{\Delta_S}}{2q_1}$, while C_1 and C_2 are complex.

F. If $\Delta_S = 0$ and $\Delta_C > 0$ we have: $S_1 = S_2 = \frac{-q_2}{2q_1}$ and $C_1 = \frac{-p_2 - \sqrt{\Delta_C}}{2p_1}$; $C_2 = \frac{-p_2 + \sqrt{\Delta_C}}{2p_1}$.

G. If $\Delta_S = 0$ and $\Delta_C < 0$ we have: $S_1 = S_2 = \frac{-q_2}{2q_1}$, while C_1 and C_2 are complex.

H. If $\Delta_S < 0$ and $\Delta_C > 0$ we have: S_1 and S_2 are complex, while $C_1 = \frac{-p_2 - \sqrt{\Delta_C}}{2p_1}$; $C_2 = \frac{-p_2 + \sqrt{\Delta_C}}{2p_1}$.

I. If $\Delta_S < 0$ and $\Delta_C = 0$ we have: S_1 and S_2 are complex, while $C_1 = C_2 = \frac{-p_2}{2p_1}$.

REFERENCES

[1] Andreas j. Steur, Niklas Bayrle (2020). S-Curves in Platform-based Business Facing the Challenge of the Tipping Point. PACIS 2020 Proceedings. 206.

[2] Jiang J, Hastings A, Lai Y-C. (2019). Harnessing tipping points in complex ecological networks. J. R. Soc. Interface 16: 20190345. Doi: 10.1098/rsif.2019.0345.

[3] Carl Boettiger, Alan Hastings (2012). Quantifying limits to detection of early warning for critical transitions. doi:10.1098/rsif.2012.0125.

[4] Junjie. J, Zi-Gang. H, Thomas P. Seager, Wei. L, Celso. G, Alan. H, Ying- Cheng. L (2017). Predicting tipping points in mutualistic networks through dimension reduction. Doi:10.1073/pnas.1714958115.

[5] Yu Meng, Celso Grebogi; (2021). Control of Tipping Points in Stochastic Mutualistic Complex Networks.

[6] Tanya Latty and Vasilis Dakos (2020). The risk of threshold responses, tipping points, and cascading failures in pollination systems.

[7] Bevo Tarika 2018. New tipping point prediction model offers insights to diminishing bee colonies.

[8] Scheffer, M., Bascompte, J., Brock, W. A., Brovkin, V., Carpenter, S. R., Dakos, V., Sugihara, G. (2009). Early-warning signals for critical transitions. Nature, 461(7260), 5359. doi: 10.1038/nature08227

[9] Dakos, V., & Bascompte, J. (2014). Critical slowing down as early warning for the onset of collapse in mutualistic communities. Proceedings of the National Academy of Sciences, 111(49), 1754617551. doi:10.1073/pnas.1406326111.

[10] Bastolla, U., Fortuna, M. A., Pascual-García, A., Ferrera, A., Luque, B., & Bascompte, J. (2009). The architecture of mutualistic networks minimizes competition and increases

biodiversity. Nature, 458(7241), 10181020. doi:10.1038/nature07950

[11] Dakos, V., & Bascompte, J. (2014). Critical slowing down as early warning for the onset of collapse in mutualistic communities. Proceedings of the National Academy of Sciences, 111(49), 1754617551. doi:10.1073/pnas.1406326111

[12] Scheffer, M., Bascompte, J., Brock, W. A., Brovkin, V., Carpenter, S. R., Dakos, V., Sugihara, G. (2009). Early-warning signals for critical transitions. Nature 461(7260): 5359. doi: 10.1038/nature08227

[13] Jackson, R. S. (2007). The Tipping Point: How Little Things Make a Big Difference. Multicultural Perspectives, 9(2), 5455. doi:10.1080/15210960701386491

[14] Peng, X., Small, M., Zhao, Y., & Moore, J. M. (2019). Detecting and Predicting Tipping Points. International Journal of Bifurcation and Chaos, 29(08), 1930022. doi:10.1142/s0218127419300222

[15] Barbier, M., Arnoldi, J. -F., Bunin, G., & Loreau, M. Generic assembly patterns in complex ecological communities. Proceedings of the National Academy of Sciences, 115(9), 21562161. (2018). doi:10.1073/pnas.1710352115

[16] Lever, J. J., van Nes, E. H., Scheffer, M., & Bascompte, J. (2014). The sudden collapse of pollinator communities. Ecology Letters, 17(3), 350359. doi: 10.1111/ele.12236.

[17] Gao, J., Barzel, B., & Barabási, A. -L. Universal resilience patterns in complex networks. Nature, 530(7590), 307312. (2016). Doi: 10.1038/nature16948.

[18] Kim, J., & Yoo, J. (2019). Platform Growth Model: The Four Stages of Growth Model. Sustainability, 11(20), 5562. doi:10.3390/su11205562

[19] Eisenmann, T.; Parker, G.; Van Alstyne, M. W. Strategies for two-sided markets. Harv. Bus. Rev. 2006, 84, 92–103.

[20] Rochet, J. C.; Tirole, J. Platform competition in two-sided markets. J. Eur. Econ. Assoc. 2003, 1, 990–1029. [CrossRef]

[21] Economides, N.; Katsamakas, E. Two-sided competition of proprietary vs. open source technology platforms and the implications for the software industry. Manag. Sci. 2006, 52, 1057–1071.

[22] Parker, G.; Van Alstyne, M. Two-sided network effects: A theory of information product design. Manag. Sci. 2005, 51, 1494–1504.

[23] Evans, D. S.; Schmalensee, R. The industrial organization of markets with two-sided platforms. Nat. Bur. Econ. Res. 2005, 3, w11603.

[24] Parker, G.; Van Alstyne, M. W.; Jiang, X. Platform ecosystems—How developers invert the firm. MIS Q. 2017, 41, 255–266.

[25] Guofu Tan, Junjie Zhou (2020). The Effects of Competition and Entry in Multi-sided Markets. The Review of Economic Studies. doi:10.1093/restud/rdaa036